1a. $x^2+5x+4=0$ it's rather easy to factor; therefore, I'll solve this equation by factoring.

M	A	N
4	5	1, 4

$(x+1)(x+4)=0$

$x+1=0$ $\quad$ $x+4=0$

$\boxed{x=-1}$ $\quad$ $\boxed{x=-4}$

$\therefore$ The solutions are -4 or -1. OR $\{-4,-1\}$

---

b) $3x^2+2x-8=0$ I, first, will check if I can factor. If I can't factor I'll use quadratic formula.

M	A	N
-24	+2	6, -4

$\rightarrow$ It works

$$\frac{(3x-4)(3x+6)}{3}=0$$

$$\frac{(3x-4)\cancel{(3)}^{1}(x+2)}{\cancel{3}_{1}}=0$$

$$(3x-4)(x+2)=0$$

$3x-4=0$ $\quad$ $x+2=0$

$3x=4$ $\quad$ $x=-2$

$x=4/3$

$\therefore$ The solutions are $\{-2, 4/3\}$

---

c. $2x^2-3x=x^2+7x$ We need to collect terms on one side. Let's do on the L.S.

$2x^2-3x-x^2-7x=0$

$x^2-10x=0$ Factoring looks to convenient for this equation.

GCF $=x$

$x(x-10)=0$

$\boxed{x=0}$ $\quad$ $x-10=0$

$\boxed{x=10}$

$\therefore \{0, 10\}$

---

d) $2(x+3)(x-4)=6x+6$ Let's expand and collect. move all terms LS

76	2
38	2
19	19

$\sqrt{2\cdot2\cdot19}$

$2(x^2-x-12)=6x+6$

$2x^2-2x-24-6x-6=0$

$2x^2-8x-30=0$ $\longrightarrow$ Let's GCF it.

$2(x^2-4x-15)=0 \longrightarrow x=\frac{-(-4)\mp\sqrt{(-4)^2-4(1)(-15)}}{2(1)}=\frac{4\mp\sqrt{76}}{2}=\frac{4\mp2\sqrt{19}}{2}=\frac{2(2\mp\sqrt{19})}{2}$

$=2\mp\sqrt{19}$ $\therefore$ The solutions are $2+\sqrt{19}$ or $2-\sqrt{19}$

2a) $1\sqrt{12} \times 2\sqrt{15}$

$= 1\cdot 2\sqrt{12\cdot 15}$

$= 2\sqrt{180}$

$$\begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$= 2\sqrt{\underline{2\cdot 2}\cdot \underline{3\cdot 3}\cdot 5}$ only the twins get out.

$= 2\cdot 2\cdot 3\cdot\sqrt{5}$

$= 12\sqrt{5}$

---

b) $2\sqrt{6}\left(2\sqrt{3} - 5\sqrt{10}\right)$ distribute $2\sqrt{6}$ over the paranthesis.

$= 2\sqrt{6}\cdot 2\sqrt{3} - 2\sqrt{6}\cdot 5\sqrt{10}$

$$\begin{array}{r|l} 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 60 & 2 \\ 30 & 2 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$= 4\sqrt{18} - 10\sqrt{60}$

$= 4\sqrt{2\cdot\underline{3\cdot 3}} - 10\sqrt{\underline{2\cdot 2}\cdot 3\cdot 5}$ only the twins get out.

$= 4\cdot 3\sqrt{2} - 10\cdot 2\sqrt{15}$

$= 12\sqrt{2} - 20\sqrt{15}$ we cannot simplify further b/c they're UNLIKE radicals.

---

c) $(3-\sqrt{2})(3\sqrt{5}+2)$ FOIL

$= 3\cdot 3\sqrt{5} + 3\cdot 2 - \sqrt{2}\cdot 3\sqrt{5} - 2\cdot\sqrt{2}$

$= 9\sqrt{5} + 6 - 3\sqrt{10} - 2\sqrt{2}$

---

d) $\dfrac{2}{3\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}$ We need to rationalize the denominator. Multiply the numerator and the denominator by $\sqrt{5}$

$= \dfrac{2\sqrt{5}}{3\cdot\sqrt{5\cdot 5}}$

$= \dfrac{2\sqrt{5}}{15}$

e) $\dfrac{2+\sqrt{5}}{3-2\sqrt{3}}$ Again, we need to rationalize the denominator. Multiply the deno. and numero by $3+2\sqrt{3}$.

$$= \frac{(2+\sqrt{5})}{(3-2\sqrt{3})} \cdot \frac{(3+2\sqrt{3})}{(3+2\sqrt{3})}$$

Notice the denominator is difference of squares. If you didn't just FOIL

$$= \frac{6+4\sqrt{3}+3\sqrt{5}+2\sqrt{15}}{9-6\sqrt{3}-6\sqrt{3}-4\sqrt{3\cdot 3}}$$

$$= \frac{6+4\sqrt{3}+3\sqrt{5}+2\sqrt{15}}{9-12}$$

$$= \frac{6+4\sqrt{3}+3\sqrt{5}+2\sqrt{15}}{-3}$$

---

3 i) $y = 2x^2 - 6x + 5$

$= 2(x^2 - 3x) + 5$ $\quad -3 \div 2 = (-1.5)$, $\quad (-1.5)^2 = 2.25$

$= 2(x^2 - 3x + 2.25 - 2.25) + 5$

$= 2(x^2 - 3x + 2.25) - 4.5 + 5$

$= 2(x - 1.5)^2 + 0.5$

a) since a is positive, graph opens up therefore, it's a min.

b) $\min = 0.5$

c) it's 1.5

d) $x = 1.5$

e) up.

---

ii) $y = -3x^2 + 4x + 20$

$= -3\left(x^2 - \frac{4}{3}x\right) + 20$ $\quad \left(\frac{-4}{3}\right) \div 2 = \frac{-4}{3} \times \frac{1}{2} = \frac{-4}{6} = -\frac{2}{3}$

$= -3\left(x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}\right) + 20$ $\quad \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$= -3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \cancel{3}^{1} \cdot \frac{4}{\cancel{9}_{3}} + 20$

$= -3\left(x - \frac{2}{3}\right)^2 + \frac{4}{3} + \frac{20 \cdot 3}{1 \cdot 3}$

$= -3\left(x - \frac{2}{3}\right)^2 + \frac{64}{3}$

a) it's a max

b) $\max = 64/3$

c) x of max is $2/3$

d) $x = 2/3$

e) down

iii) $y = 4x^2 - 10x - 1$

$= 4(x^2 - 2.5x) - 1$ $\quad -2.5 \div 2 = -1.25$

$(-1.25)^2 = 1.5625$

$= 4(x^2 - 2.5x + 1.5625 - 1.5625) - 1$

$= 4(x^2 - 2.5x + 1.5625) - 6.25 - 1$

$= 4(x - 1.25)^2 - 7.25$

a) it's a min

b) $y = -7.25$

c) $x = 1.25$

d) $x = 1.25$

e) up.

---

4. Determine the equation of the quadratic function in the form $y = ax^2 + bx + c$ that passes through the point (2, 7) and has zeros of 3 and – 4.

Question is providing us the x-int ; therefore, we can construct the equation is factored form.

step1 $y = a(x-r)(x-s)$ $\quad r = 3 \quad s = -4 \quad (\overset{x}{2}, \overset{y}{7})$

$7 = a(2-3)(2-(-4))$

$7 = a(-1)(6)$

$7 = -6a$

$\boxed{-7/6 = a}$

Step2 $y = \frac{-7}{6}(x-3)(x+4)$ $\quad$ FOIL

$= \frac{-7}{6}(x^2 + x - 12)$ $\quad$ distribute $-7/6$

$y = \frac{-7}{6}x^2 - \frac{7}{6}x + 14$

5. Solve the system of equations using an algebraic method.

$y = 3x^2 - 2x - 1$

$y = -x - 6$

$3x^2 + 2x - 1 = -x - 6$ $\quad$ equal each expression, then collect terms on one side.

$3x^2 + 2x - 1 + x + 6 = 0$

$3x^3 + 3x + 5 = 0$ $\quad$ We need to use the formula

$x = \frac{-3 \mp \sqrt{(3)^2 - 4(3)(5)}}{2(3)}$

$= \frac{-3 \mp \sqrt{-51}}{6}$ → Since the discriminant is less than 0, no solution to this system. In other words, these graphs do not intersect.

6. For what values of k will the function $y = kx^2 - 4x + k$ have no zeros?

It means the discriminant will be less than "0"

$b^2 - 4ac < 0$ $\quad a = k \quad b = -4 \quad c = k$

$(-4)^2 - 4(k)(k) < 0$

$16 - 4k^2 < 0$ move $-4k^2$ to the other side

$\frac{16}{4} < \frac{4k^2}{4}$ divide each side by 4

$y = 3x^2 - 4x + 3$

$\sqrt{4} < \sqrt{k^2}$ square root each side

$2 < |k| \rightarrow 2 < k$

$2 < |k| \rightarrow 2 < -k \Rightarrow -2 > k$ (when we divide each side by $-$, Inequality switch)

Check

$k = 3$ $\quad y = 3x^2 - 4x + 3$

$b^2 - 4ac \overset{?}{<} 0$

$(-4)^2 - 4(3)(3)$

$16 - 36$

-20 ✓

or

$k = -3$ $\quad y = -3x^2 - 4x - 3$

$b^2 - 4ac \overset{?}{<} 0$

$(-4)^2 - 4(-3)(-3) < 0$

$16 - 36 < 0$ ✓

$-20 < 0$

$\therefore k > 2$ or $k < -2$

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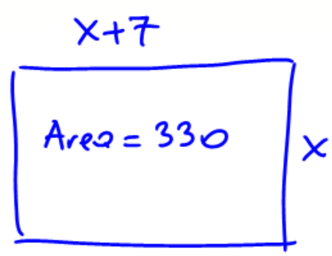
7. A rectangle has an area of 330m^2. One side is 7 metres longer than the other side. What are the dimensions of the rectangle?



Let "x" be one side

$A = L \cdot W$

$330 = x(x+7)$ expand & collect

$0 = x^2 + 7x - 330$

$x = \frac{-7 \mp \sqrt{49 - 4(-330)}}{2}$

$= \frac{-7 \mp \sqrt{1369}}{2}$

$= \frac{-7 \mp 37}{2}$

$x_1 = \frac{-7 + 37}{2} = 15$

$\cancel{x_2 = \frac{-7 - 37}{2} = -22}$

$\therefore$ The dimensions are 22m and 15m.

8. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, y, in metres, t seconds after jumping can be modelled by

$$y_1 = -4.9t^2 + t + 360$$
$$y_2 = -4t + 142$$

How long after jumping did the daredevil release his parachute?

$-4.9t^2 + t + 360 = -4t + 142$

$-4.9t^2 + t + 360 + 4t - 142 = 0$

$-4.9t^2 + 5t + 218 = 0$

$x = \dfrac{-5 \mp \sqrt{(5)^2 - 4(4.9)(218)}}{2(-4.9)}$

$= \dfrac{-5 \mp \sqrt{4298.8}}{-9.8}$

$= \dfrac{-5 \mp 65.6}{-9.8}$

$x_1 = \dfrac{-5 + 65.6}{-9.8} = -6.2$ sec.

$x_2 = \dfrac{-5 - 65.6}{-9.8} = 7.2$ sec.

$\therefore$ After 7.2 sec.

9. The population of a region can be modelled by the function $y = 0.4t^2 + 10t + 50$, where y is the population in thousands and t is the time in years since the year 1995.

a. What was the population in 1995?

b. What will be the population in 2010?

a) $t = 0$, $y = 50$ $\therefore$ 50,000 was the population.

b) $t = 2010 - 1995$

$= 15$ years

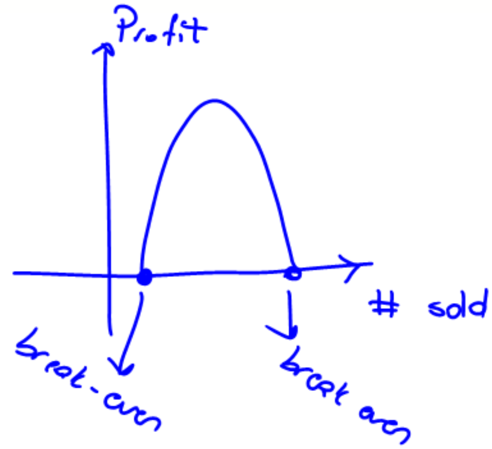
$y = 0.4(15)^2 + 10(15) + 50$

$= 290$

$\therefore$ It'll be 290,000.

10. The profit function for a new product is given by $y = -4x^2 + 28x - 40$, where x is the number sold in thousands. How many items must be sold for the company to break even?

We need to find "x" intercepts.



$y = -4x^2 + 28x - 40$ Check GCF first

$y = -4(x^2 - 7x + 10)$

$y = -4(x-2)(x-5)$

$x - 2 = 0$ $\quad$ $x - 5 = 0$

$\boxed{x = 2}$ $\quad$ $\boxed{x = 5}$

$\therefore$ Either 2000 or 5000 items must be sold.

11. It costs a bus company \$225 to run a minibus on a ski trip, plus \$30 per passenger. The bus has seating for 22 passengers, and the company charges \$60 per fare if the bus is full. For each empty seat, the company has to increase the ticket price by \$5. How many empty seats should the bus run with to maximize profit from this trip?

Let "x" rep # of empty seats.

step Profit = Revenue − Cost

step Revenue = (Price) · (Amount of passengers)

$= (60+5x)(22-x)$

Cost $= 225 + 30(22-x)$

$P = (60+5x)(22-x) - [225 + 30(22-x)]$ expand

$= 1320 - 60x + 110x - 5x^2 - 225 - 660 + 30x$

$= -5x^2 + 80x + 435$ Let's complete the square

$= -5(x^2 - 16x) + 435$ $\quad -16 \div 2 = -8$

$(-8)^2 = 64$

$= -5(x^2 - 16x + 64 - 64) + 435$

$= -5(x^2 - 16x + 64) + 320 + 435$

$= -5(x-8)^2 + 755$

$\therefore$ The bus should run with 8 empty seats to have max profit of \$755.

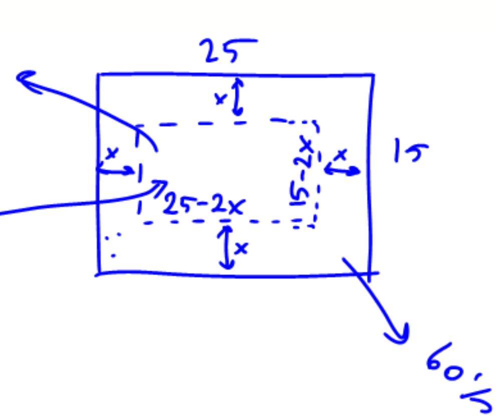
12. Andrew mows a strip of uniform width around his 25m by 15m rectangular lawn that is 60% of the original area. What is the width of the strip?

Area mown is 60% of the total Area

then area untouched is 40% of the total area

$= 0.40 \times (25)(15)$

$= 150 m^2$



$150 = (25-2x)(15-2x)$ FOIL, then collect

$150 = 375 - 50x - 30x + 4x^2$

$0 = 4x^2 - 80x + 225$

$x = \dfrac{-(-80) \mp \sqrt{(-80)^2 - 4(4)(225)}}{2(4)}$

$= \dfrac{80 \mp \sqrt{2800}}{8}$

$= \dfrac{80 \mp 52.9}{8}$

$\dfrac{80 + 52.9}{8} \doteq 16.6$ x cannot equal 16.6

$\dfrac{80 - 52.9}{8} \doteq 3.4$

$\therefore$ The width is 3.4 m.

13. If $y = x^2 - 6x + 14$ and $y = -x^2 - 20x - k$, determine the value of k so that there is exactly one point of intersection between the two parabolas.

$x^2 - 6x + 14 = -x^2 - 20x - k$

$x^2 - 6x + 14 + x^2 + 20x + k = 0$

$2x^2 + 14x + (14 + k) = 0$

In order for one solution to happen, the discriminant must equal "0".

$b^2 - 4ac = 0$

$(14)^2 - 4(2)(14+k) = 0$

$196 - 8(14+k) = 0$

$196 - 112 - 8k = 0$

$\dfrac{84}{8} = \dfrac{8k}{8}$

$\boxed{k = 10.5}$

$\therefore$ k must be 10.5.