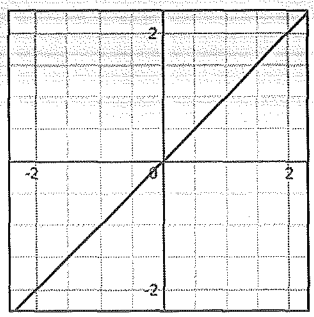
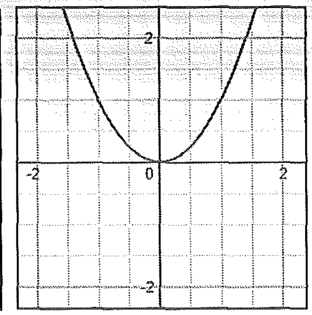
Day 3: Combined Function Applications

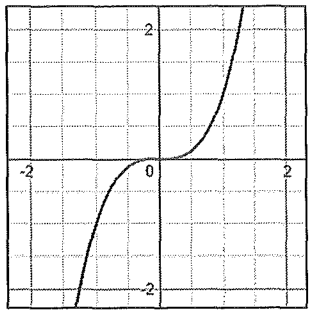
Warm Up

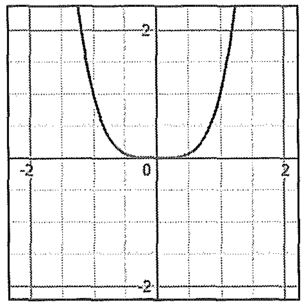
Consider the power functions $f(x)$, $g(x)$, $h(x)$, and $i(x)$ and their combinations listed in the table below. Complete the following table:

Function	Is the function even, odd, or neither?
$f(x) = x$	odd
$g(x) = x^2$	even
$h(x) = x^3$	odd
$i(x) = x^4$	even
$f(x)g(x) = x^3$	odd
$\frac{f(x)}{g(x)} = \frac{1}{x}$	odd
$\frac{g(x)}{f(x)} = x$	odd
$f(x)h(x) = x^4$	even
$g(x)i(x) = x^6$	even
$\frac{h(x)}{f(x)} = \frac{x^3}{x} = x^2$	even
$\frac{i(x)}{g(x)} = \frac{x^4}{x^2} = x^2$	even









Even and Odd Symmetry of Product and Quotient Functions

When functions are multiplied or divided, their symmetry determines the symmetry of the product or quotient function.

Products
$Even \cdot Even =$ even
$Odd \cdot Odd =$ even
$Even \cdot Odd =$ odd

Quotients
$\frac{Even}{Even} =$ even
$\frac{Odd}{Odd} =$ even
$\frac{Even}{Odd} =$ odd
$\frac{Odd}{Even} =$ odd

Example One – Even and Odd Symmetry

Given $f(x) = 2\sin x$ and $g(x) = |x|$, determine the symmetry of $f(x)g(x)$

$h(x) = f(x)g(x) = (2\sin x)(|x|$

$h(-x) = 2\sin(-x)\,|-x| = -2\sin x\,|x| = -h(x)$

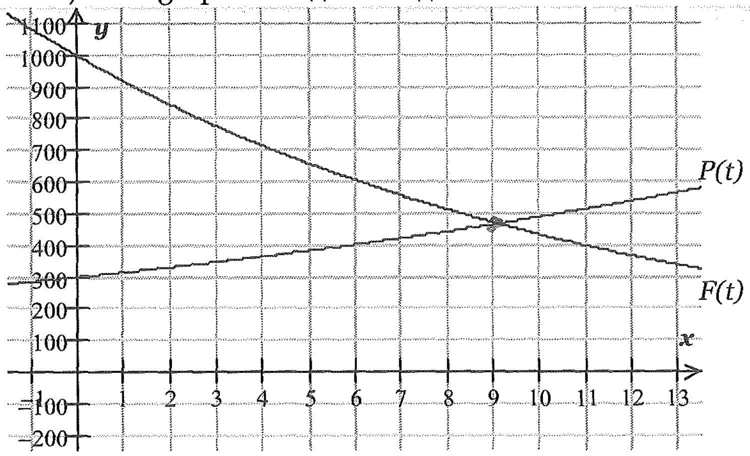
$\therefore$ $h(x)$ is an odd function.

Example Two – Applications of Combined Functions

A fish pond initially has a population of 300 fish.
The fish population, $P(t)$, grows as a function of time, t years as $P(t) = 300(1.05)^t$.

The pond initially has 1000 units of food, where 1 unit can sustain one fish for a year.
The amount of fish food, $F(t)$, is decreasing according to the function $F(t) = 1000(0.92)^t$.

a) The graphs of $P(t)$ and $F(t)$ are shown below. Describe the shape of each exponential function.



P(t) growth by 5%
initial P = 300

F(t) is decaying by 8%
initial population: 1000.

a) State the **domain and range** for each function.

P(t)	*F(t)*
D: $\{t \in \mathbb{R} \mid t \geq 0\}$	$\{t \in \mathbb{R} \mid t \geq 0\}$
R: $\{y \in \mathbb{R} \mid y \geq 300\}$	$\{y \in \mathbb{R} \mid y \leq 1000\}$

b) From the graph, state the **point of intersection** for the two functions.

$(9, 465)$

c) Using the equations, verify the point of intersection **algebraically**. *Hint: use logs*

$$300(1.05)^t = 1000(0.92)^t$$

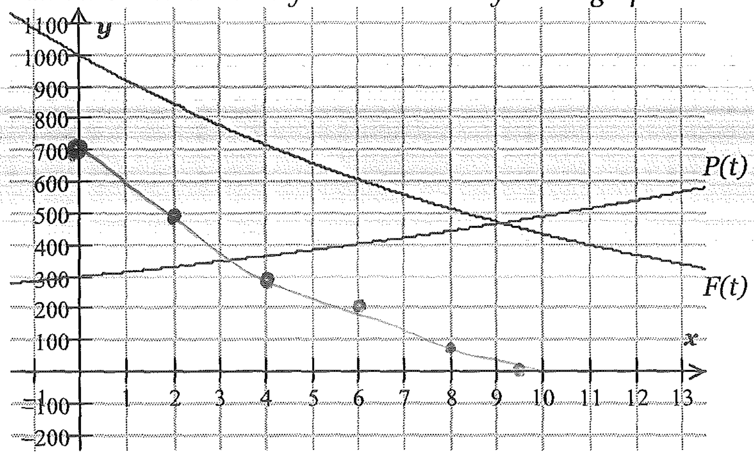
$$\left(\frac{1.05}{0.92}\right)^t = \frac{1000}{300}$$

$$t = \frac{\log\left(\frac{1000}{300}\right)}{\log\left(1.05/0.92\right)} = 9.1$$

$P(9.1) \doteq 468$

$\therefore$ POI is $(9.1, 468)$

d) Use the superposition principle to graph $S(t) = F(t) - P(t)$ on the same set of axes.
Hint: use x-values that you can see visibly on the graph.



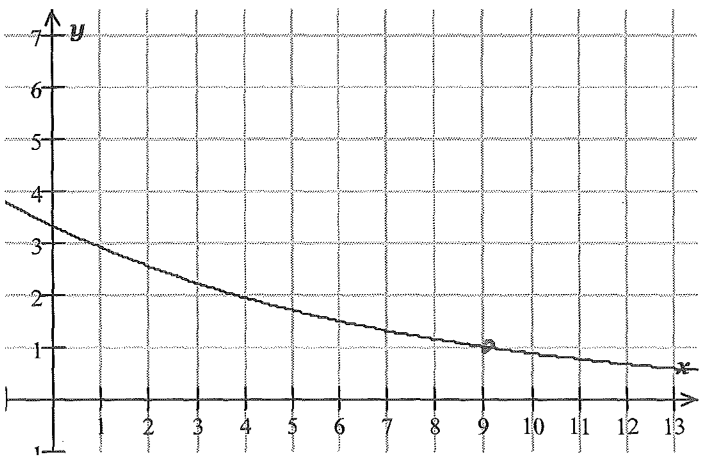
e) What does the function $S(t)$ represent?

How much food units are present that is extra (NOT USED)

f) From the graph, determine the **t-intercept** for $S(t)$. 9.1 = t
Why is this point called the **crisis point**?

↳ After 9.1, the population of fishes is more than food remaining. Hence, fishes may not survive.

g) The graph of $R(t) = \dfrac{F(t)}{P(t)}$ is shown at the right.



What does this function represent?

how much units of food for one fish is present.

h) Describe the **shape** of this function.

decaying graph.

i) What are the **coordinates** of this function at the crisis point? Explain the meaning of this point.

(9.1, 1) Since after 9.1, less than one unit present

j) Describe the living conditions of the fish population **before**, **at** and **after** the crisis point.

before 9.1: extra food units present

at 9.1: just enough food units present for fishes to survive.

after 9.1: not enough food present. fishes may not survive