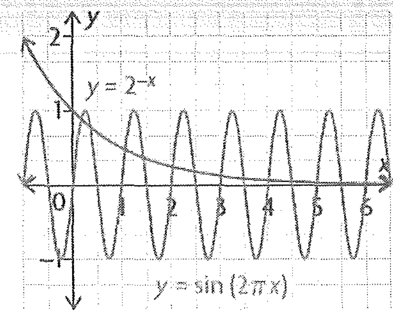
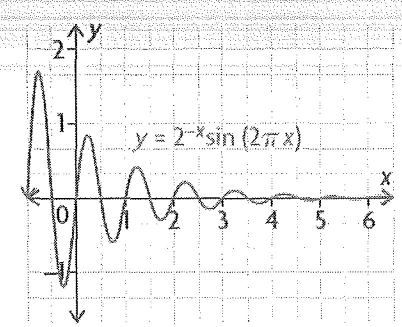
Day 2 – Product and Quotient of Functions

Have you ever wondered how sound engineers cause the music to fade out, gradually, at the end of a song? The music fades out because the sine waves that represent the music are being squashed or **damped**. Mathematically, this can be done by multiplying a sine function by another function.

The functions defined by $g(x) = \sin(2\pi x)$ and $f(x) = 2^{-x}$ are shown. Observe what happens when these functions are multiplied together to produce the graph of $y = 2^{-x}\sin(2\pi x)$





- A combined function in the form $y = f(x)g(x)$ represents the product of two functions, $f(x)$ and $g(x)$. To find the product, multiply the y coordinates.

- A combined function in the form $y = \dfrac{f(x)}{g(x)}$ represents the quotient of two functions, $f(x)$ and $g(x)$, for $g(x) \neq 0$. To find the quotient, divide the y coordinates.

- The domain of the product or quotient of functions is the domain common to the component functions.

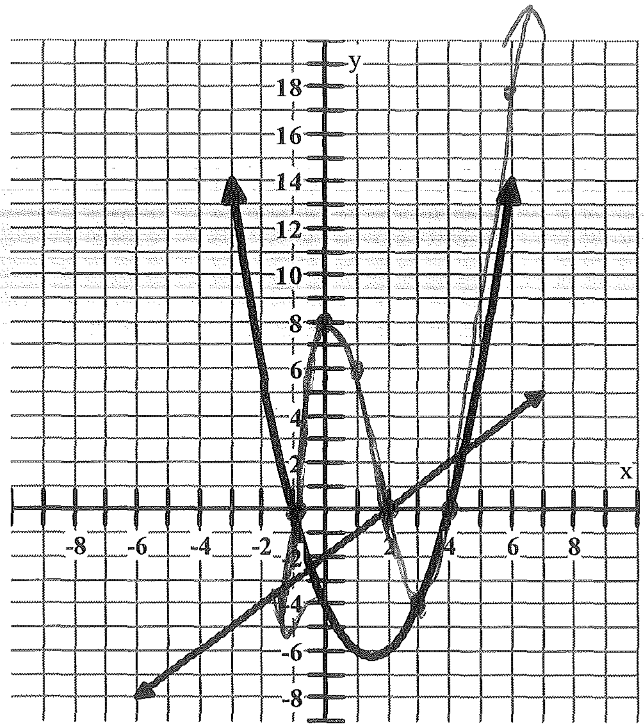
- The domain of a quotient function is further restricted by excluding any values that make the denominator equal to zero. **(Need to state these restrictions)**

→ when is there a hole in the graph?
when there is a common factor in the numerator and denominator.

→ when is there a VA?
when there is a factor in the denominator that can = 0. eg $\frac{1}{x^2+1}$ doesnt have VA

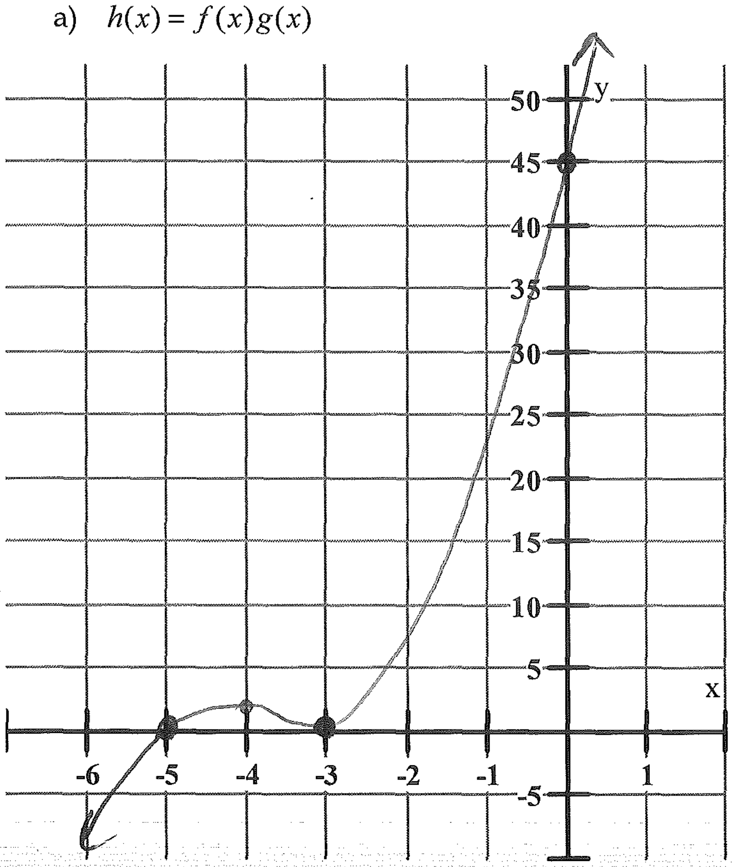
EX 1 – Given $f(x) = x^2 - 3x - 4$ and $g(x) = x - 2$, graph the function $h(x) = f(x)g(x)$ using the superposition principal.

x	$f(x)$	$g(x)$	$h(x) = f(x)g(x)$
-2	6	-4	-24
-1	0	-3	0
0	-4	-2	8
1	-6	-1	6
2	-6	0	0
3	-4	1	-4
4	0	2	0
5	6	3	18



EX 2 – Given $f(x) = x + 3$ and $g(x) = x^2 + 8x + 15$, sketch a graph of the following combined functions and then state its domain and range.

a) $h(x) = f(x)g(x)$



$h(x) = (x+3)(x^2+8x+15)$

$= (x+3)(x+5)(x+3)$

$= (x+3)^2(x+5)$

y-int: 45

$x \to \infty \quad y \to \infty$

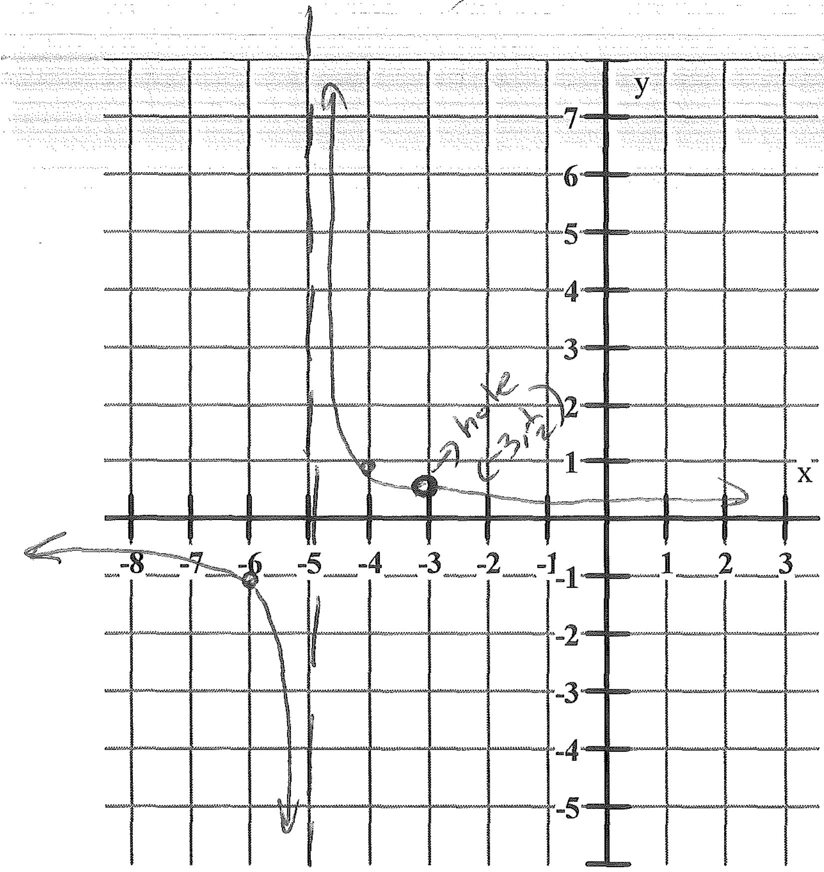
$x \to -\infty \quad y \to -\infty$

NOTE: $(-4, 1)$ is a point on $h(x)$.

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$ ← cubic

b) $j(x)=\dfrac{f(x)}{g(x)} = \dfrac{\cancel{(x+3)}}{\cancel{(x+3)}(x+5)} = \dfrac{1}{x+5}$ hole at $(-3,\frac{1}{2})$



Domain: $\{x\in\mathbb{R} \mid x\neq -3,-5\}$

Range: $\{y\in\mathbb{R} \mid y\neq 0, 1/2\}$

HA   hole

Homework:

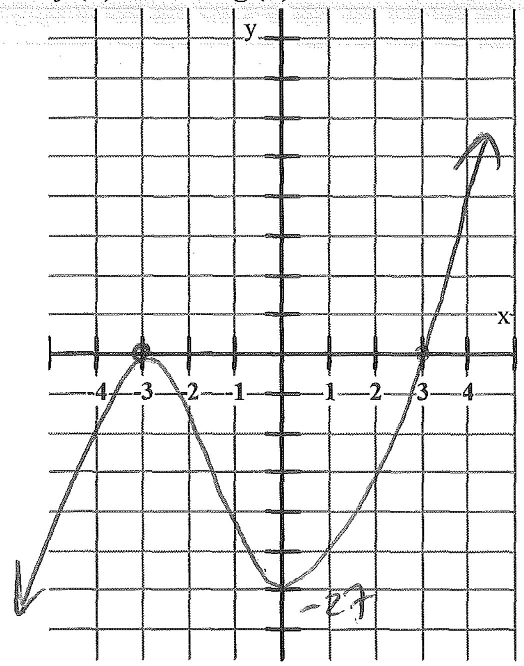
Practice Questions: Given $f(x)$ and $g(x)$, sketch a graph of the following combined functions and then state its domain and range. *Complete your side work on a separate piece of paper.*

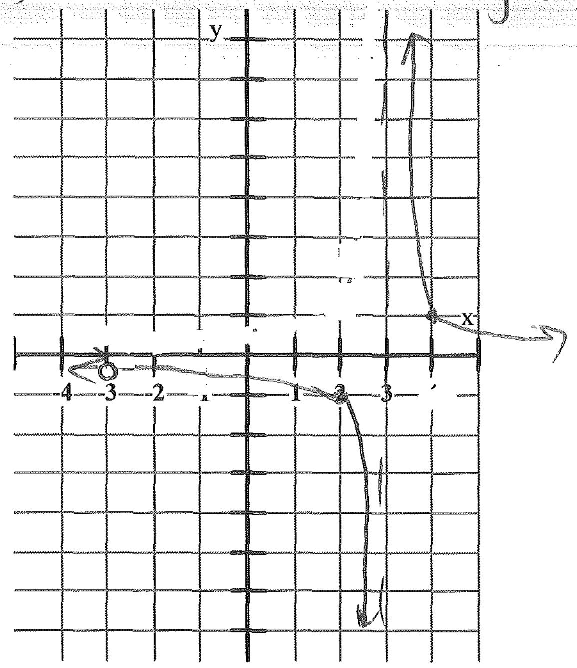
a) $h(x) = f(x)g(x)$ b) $j(x) = \frac{f(x)}{g(x)}$

hole $(-3, -\frac{1}{6})$

1. $f(x) = x + 3$, $g(x) = x^2 - 9$ $h = (x+3)^2(x-3)$

$j(x) = \frac{1}{x-3}$





Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$

Domain: $\{x \in \mathbb{R} \mid x \neq \pm 3\}$

Range: $\{y \in \mathbb{R} \mid y \neq -1/6, 0\}$

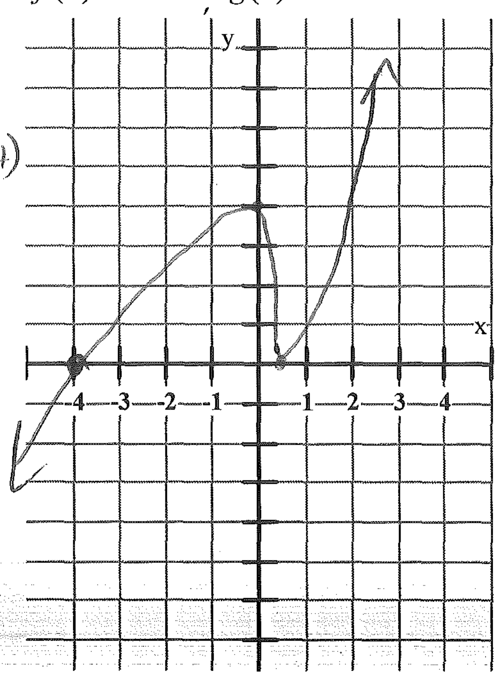
2. $f(x) = 2x - 1$, $g(x) = 2x^2 + 7x - 4$

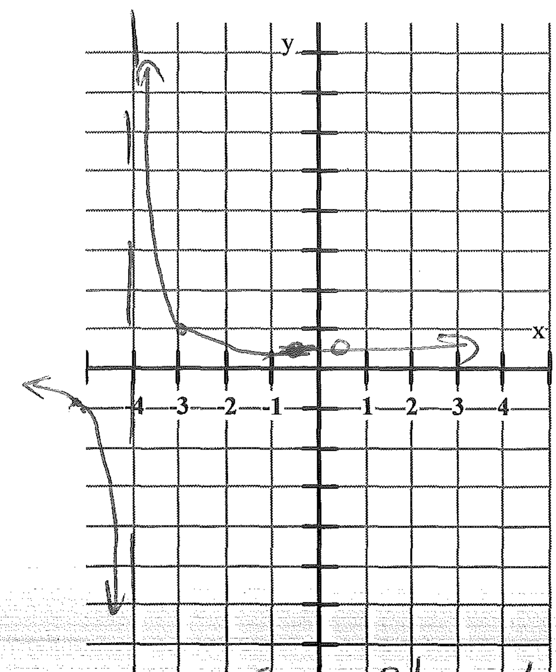
$h(x) = (2x-1)^2(x+4)$

$j(x) = \frac{2x-1}{(2x-1)(x+4)}$

$= \frac{1}{x+4}$

hole $(\frac{1}{2}, \frac{2}{9})$





Domain:

Range:

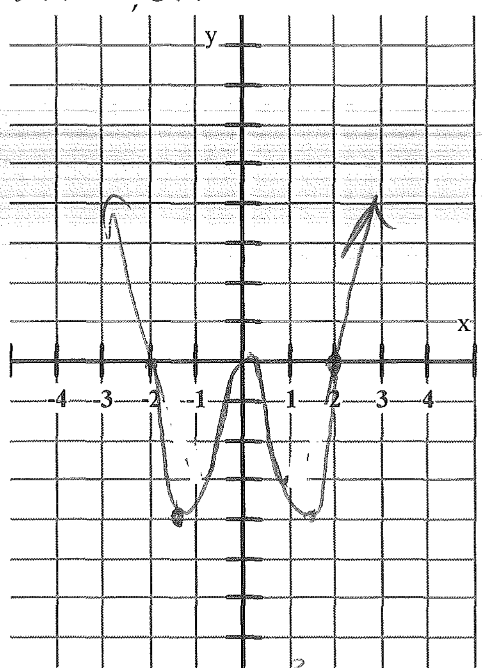
Domain: $\{x \in \mathbb{R} \mid x \neq -4, \frac{1}{2}\}$

Range: $\{y \in \mathbb{R} \mid y \neq 0, \frac{2}{9}\}$

$x^2(x-2)(x+2)$

$y = \dfrac{\cancel{x}}{\cancel{x}(x-2)(x+2)}$

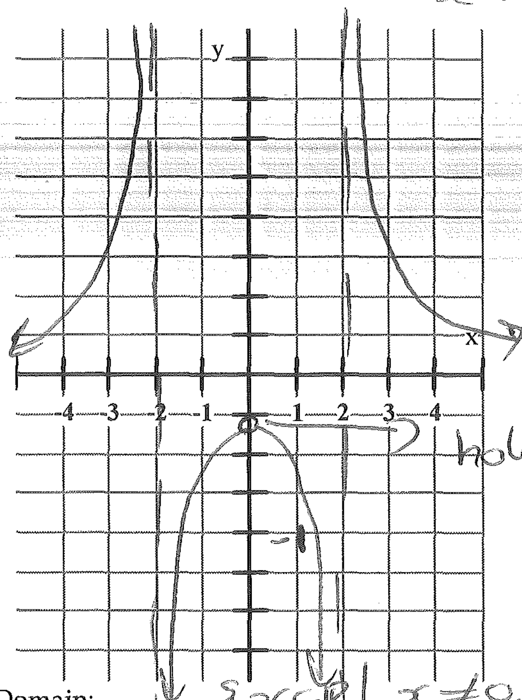
3. $f(x) = x$, $g(x) = x^3 - 4x$



Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \geq -4\}$

If calculus concepts used, critical point (local min) occur at $(\sqrt{2}, -4)$ $(-\sqrt{2}, -4)$



hole $(0, \frac{-1}{4})$

Domain: $\{x \in \mathbb{R} \mid x \neq 0, \pm 2\}$

Range: $\{y \in \mathbb{R} \mid y > 0, \; y < \frac{-1}{4}\}$