Unit 5 Mathematical Models - Review

1. Examine each equation and identify which type of relation (linear, quadratic, or exponential) each equation represents. Explain how you know.

a) $y = 1.3(1.7)^x$

Exponential
↳ x is in the exponent

b) $y = 2x^2 - x + 4$

Quadratic
↳ The highest power is 2

c) $y = 5 - 2x$

Linear
↳ no exponents

2. Determine if each table of values represents a linear, quadratic, or exponential model. Explain how you know.

Time (days)	Volume (mL)
0	3.15
2	6.05
4	8.95
6	11.85
8	14.75
10	17.65

+2.9, +2.9, +2.9, +2.9, +2.9

Linear
↳ First differences are constant.

Year	Population
2005	423
2007	648
2009	993
2011	1521
2013	2330

$648 \div 423 = 1.53$
$993 \div 648 = 1.53$
$1521 \div 993 = 1.53$
$2330 \div 1521 = 1.53$

Exponential
↳ Values differ by a constant multiple

x	y
1	23
3	55
5	103
7	167

32, 48, 64
16, 16

Quadratic
↳ Second differences are constant

3. For the **linear** table of values above, find the **slope.** Include units.

Pick 2 points from the table: (0, 3.15) (2, 6.05)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.05 - 3.15}{2 - 0} = \frac{2.9}{2} = 1.45 \frac{\text{mL}}{\text{day}}$

4. Simplify each expression (expression each as a power with positive exponents).

a) $\dfrac{(3^{-2})(3^3)}{3^{-1}}$

$= \dfrac{3^1}{3^{-1}}$

$= 3^2$

b) $\dfrac{(-3)^4 \times (-3)^5}{\left[(-3)^3\right]^4}$

$= \dfrac{(-3)^9}{(-3)^{12}}$

$= (-3)^{-3}$

$= \dfrac{1}{(-3)^3}$

c) $\dfrac{p^{-4}q^3}{p^2q^{-2}}$

$= p^{-4-2}q^{3-(-2)}$

$= p^{-6}q^5$

$= \dfrac{q^5}{p^6}$

d) $\left(u^2v^0w^{-1}\right)^{-2}$

$= u^{-4}v^0w^2$

$= u^{-4}w^2$

$= \dfrac{w^2}{u^4}$

5. Write as a root, then evaluate [4 marks]

a) $125^{\frac{2}{3}} = \sqrt[3]{125^2}$

$= 5^2$

$= 25$

b) $(+256)^{\frac{3}{8}} = \sqrt[8]{(+256)^3}$

$= 2^3$

$= 8$

6. Solve for *x*.

a) $10^{-3x+1} = 10^{2x-4}$

b) $3^{3x+1} = 9^{x-2}$

c) $4^{x-3} = 8^{x+1}$

d) $27^2 = 3^{2x+1}$

e) $3^{2x+3} = \frac{1}{9}$

a) $10^{-3x+1} = 10^{2x-4}$

$-3x+1 = 2x-4$

$-3x-2x = -4-1$

$-5x = -5$

$x = \frac{-5}{-5}$

$x = 1$

b) $3^{3x+1} = 9^{x-2}$

$3^{3x+1} = (3^2)^{x-2}$

$3^{3x+1} = 3^{2x-4}$

$3x+1 = 2x-4$

$3x-2x = -4-1$

$x = -5$

c) $4^{x-3} = 8^{x+1}$

$(2^2)^{x-3} = (2^3)^{x+1}$

$2^{2x-6} = 2^{3x+3}$

$2x-6 = 3x+3$

$2x-3x = 3+6$

$-x = 9$

$x = -9$

d) $27^2 = 3^{2x+1}$

$(3^3)^2 = 3^{2x+1}$

$3^6 = 3^{2x+1}$

$6 = 2x+1$

$6-1 = 2x$

$5 = 2x$

$\frac{5}{2} = x$

e) $3^{2x+3} = \frac{1}{9}$

$3^{2x+3} = 3^{-2}$

$2x+3 = -2$

$2x = -2-3$

$2x = -5$

$x = -\frac{5}{2}$

7. The formula $A = P(1+i)^n$ can be used to model the growth of money when interest is compounded monthly. Solve for i.

$A = P(1+i)^n$

$\frac{A}{P} = (1+i)^n$

$\sqrt[n]{\frac{A}{P}} = 1+i$

$\sqrt[n]{\frac{A}{P}} - 1 = i$

8. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. Solve for r.

$V = \frac{4}{3}\pi r^3$

$3V = 4\pi r^3$

$\frac{3V}{4\pi} = r^3$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

9. The formula $E = mc^2$ related the mass of an object (m), the speed of light (c) and energy (E).

- Solve for m.
- Solve for c.

$E = mc^2$

$\frac{E}{c^2} = m$

$E = mc^2$

$\frac{E}{m} = c^2$

$\sqrt{\frac{E}{m}} = c$

10. Solve for x to two decimal places using a table of values to guess and check

$$4^x = 300$$

x	4^x
2	16
3	64
4	256
5	1024
4.5	512
4.2	337.79
4.1	294.07
4.15	315.17
~~4.12~~	~~302.33~~
4.11	298.17

$\therefore x = 4.11$

11. Cynthia deposits money in a high interest savings account. The value of the account, V dollars, after t years is given by the equation:

$$V = 2000(1.04)^t$$

a) What does 2000 represent?
b) What does 1.04 represent?
c) How much money is the account after 13 years?
d) Cynthia will buy a used car when she has saved $5000. After how many years will Cynthia buy her car?

a) 2000 represents the amount of money deposited at the beginning (The initial value = $2000)

b) 1.04 is the growth rate (0.04 = 4% more money each year)

c) $V = 2000(1.04)^{13}$
$= 2000(1.66507)$
$= \$3330.15$

d) $5000 = 2000(1.04)^t$
$\frac{5000}{2000} = 1.04^t$
$2.5 = 1.04^t$

t	1.04^t
2	1.0816
5	1.21665
10	1.48
20	2.19
25	2.6658
22	2.3699
23	2.4647
24	2.5633

Cynthia must wait 23 years to buy her car.

12. Tritium, a radioactive gas that builds up in CANDU nuclear reactors, is collected, stored in pressurized gas cylinders, and sold to research laboratories. Tritium decays into helium over time. Its half-life is about 12.3 years.
    a) Write an equation that gives the mass of tritium remaining in a cylinder that originally contained 500 g of tritium.
    b) Estimate the time it takes until less than 5 g of tritium is present.

a) Radioactive decay: $A = A_0(0.5)^{t/h}$

$$\boxed{A = 500(0.5)^{t/12.3}}$$

b) $A = 5$

$5 = 500(0.5)^{t/12.3}$

$\frac{5}{500} = (0.5)^{t/12.3}$

$0.01 = (0.5)^{t/12.3}$

t	$0.5^{t/12.3}$
5	0.7579
50	0.0625
60	0.0359
70	0.0206
80	0.0118
85	0.00897
82	0.010598
83	0.0100267
84	0.00948

∴ It will take approximately 83 years

13. A colony of ~~bacteria doubles~~ in size every 20 min. How long will it take for a colony of 20 bacteria to grow to a population of 10000? ↳ Exponential growth

d (20 min), A_0 (20), A (10000)

$A = A_0 2^{t/d}$

$A = 20(2)^{t/20}$

$1000 = 20(2)^{t/20}$

$\frac{10000}{20} = 2^{t/20}$

$500 = 2^{t/20}$

t	$2^{t/20}$
100	32
200	1024
150	181.
180	512
175	430.5
178	477.7
179	494.56

∴ It will take 179 minutes.

Don't forget: $A = A_0(2)^{\frac{t}{d}}$ $A = A_0(0.5)^{\frac{t}{h}}$