# Day 8: 5.5 Instantaneous Rates of Change

**EX 1 -** The height, $h$, in metres, of a car above the ground as a Ferris wheel turns can be modelled by the function $h(t) = 18\sin\frac{\pi}{90}t + 20$, where $t$ is the time, in seconds.

**a)** Determine the **average rate of change** of $h$ in the following time intervals, rounded to five decimal places.

| Interval | $\Delta t$ | $\Delta h(t)$ | Average rate of change of $h(t)$ (m/s) |
|---|---|---|---|
| 7 s to 8 s | 1 | $= h(8) - h(7)$<br>$= \left[18\sin\frac{8\pi}{90} + 20\right] - \left[18\sin\frac{7\pi}{90} + 20\right] =$<br>$= 0.60688$ | 0.6088 m/s |
| 7.9 s to 8 s | 0.1 | $= \left[18\sin\frac{8\pi}{90} + 20\right] - \left[18\sin\frac{7.9\pi}{90} + 20\right]$<br>$= 0.06043$ | $\frac{0.06043}{0.1} = 0.60428$ m/s |
| 7.99 s to 8 s | 0.01 | $= \left[18\sin\frac{8\pi}{90} + 20\right] - \left[18\sin\frac{7.99\pi}{90} + 20\right]$<br>$= 0.00604$ | 0.6040 m/s |

b) Estimate a value for the instantaneous rate of change of $h$ at $t = 8$ s.

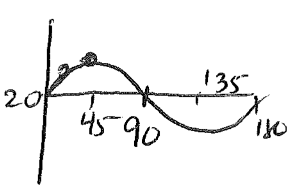
IROC = 0.60 m/s

c) What physical quantity does this instantaneous rate of change represent in this case?

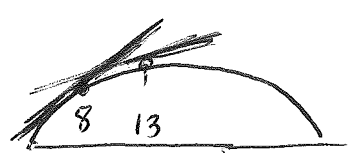
At time 8 seconds, the height increasing by 0.60 m/s.
(The Ferris wheel is going up)

d) Would you expect the instantaneous rate of change for h(t) be the same at $t = 13$ s? Explain.

$h(t) = 18\sin\frac{\pi t}{90} + 20$ $\left[\text{Period} = \frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{90}} = 180\right]$

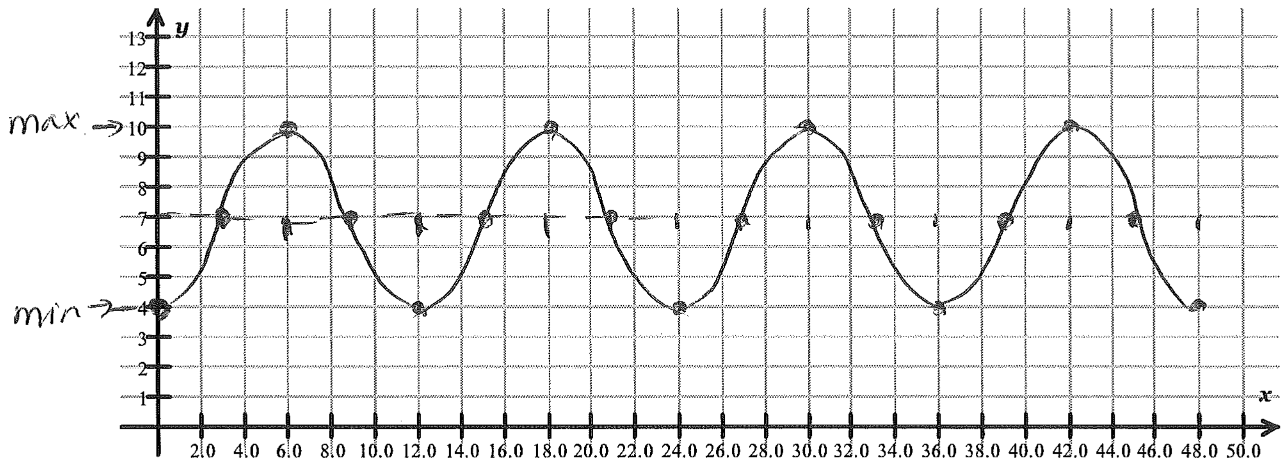



At time = 13 seconds, the IROC would be lower (different)



**EX 2** - The water depth near a harbour varies throughout the day. A camper observes that the water depth is 10 m at high tide and 4 m at low tide and that one tide cycle occurs approximately every 12 hours.

a) Graph the water depth, d(t) in metres, versus time, t in hours, for the **next 48 hours**. Assume that you start measuring at low tide which occurs at midnight.




$a=3 \quad p=12 \quad k=\frac{2\pi}{12}=\frac{\pi}{6}$

b) Write a **cosine** equation to represent the function. $d(t)=-3\cos\frac{\pi}{6}t+7$

c) Based on your equation, what is the **water depth at 10 am**?

$d(10)=-3\cos\frac{10\pi}{6}+7$

$=5.5m$

d) Complete the table below to find the instantaneous rate of change of the water depth relative to time for **t = 3**.

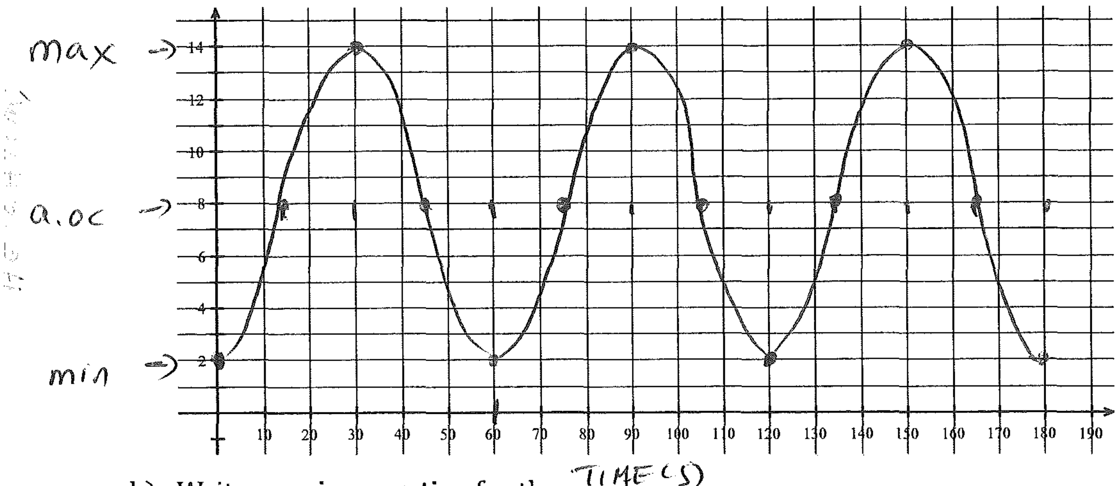
| Intervals | Δt | Δd | Δd/Δt |
|---|---|---|---|
| $3 \leq t \leq 3.1$ | 0.1 | $d(3.1)-d(3)$ $=0.1570$ | $\frac{0.1570}{0.1}=1.5700$ m/s |
| $3 \leq t \leq 3.01$ | 0.01 | $d(3.01)-d(3)$ $=0.015708$ | 1.5708 m/s |
| $3 \leq t \leq 3.001$ | 0.001 | $d(3.001)-d(3)$ $=0.0015708$ | 1.5708 m/s |

∴ The instantaneous rate of change at t = 3 is <u>1.6</u> *(round to 1 decimal)*
m/s

Homework Questions:

1. A carnival Ferris wheel with a radius of 6 m makes one complete revolution every 60 seconds. The bottom of the wheel is 2 m above the ground. $a=6$ $p=60$ $k=\frac{2\pi}{60}=\frac{\pi}{30}$ $min=2$ $max=14$

a) Graph a rider's height above the ground, $h(t)$ in metres, as a function of time, $t$ in seconds, for three revolutions. Assume that the riders boards at the bottom of the Ferris wheel. *Label the axes.*




b) Write a **cosine equation** for the graph.

$h(t) = -6\cos\left(\frac{\pi}{30}t\right)+8$

c) Calculate the rider's height above the ground after **10 seconds**.

$h(10) = 5$ m

d) Write a new **cosine equation** for the graph if the Ferris wheel completes one revolution every 30 seconds.

$h(t) = -6\cos\left(\frac{\pi}{15}t\right)+8$

e) State the time(s) when the instantaneous rate of change is a maximum, and estimate this rate of change:

$t = 15, 75, 135$ seconds. $IROC = \frac{h(15.01)-h(15)}{0.01} \doteq 0.63$ m/s

f) State the time(s) when the instantaneous rate of change is a minimum, and estimate this rate of change:

$t = 45, 105, 165$ seconds. $IROC = -0.63$ m/s

g) State the time(s) when the instantaneous rate of change is zero.

At local max/min: $t = 0, 30, 60, 90, 120, 150, 180$

$IROC = 0 \Rightarrow$ tangent is horizontal.