# Day 7: Solving Quadratic Equations.

**To solve a quadratic trig equation algebraically:**
- Factor (if possible)
- Isolate for $\sin x$, $\cos x$, *or* $\tan x$
  - To find an exact solution: use unit circle
  - To find an approximate solution: use calculator & CAST

**EX 1 - Warm up:** Factor each of the following trig expressions.

a) $\cos^2\theta - \cos\theta$

$= \cos\theta(\cos\theta - 1)$

b) $\sin^2\theta - 2\sin\theta + 1$

$= (\sin\theta - 1)(\sin\theta - 1)$

$= (\sin\theta - 1)^2$

c) $25\tan^2 x - 100$

$= 25(\tan x - 2)(\tan x + 2)$

**EX 2** – Determine the exact solutions for the following trigonometric equations in the interval $[0, 2\pi]$

a) $2\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{1/2} = \pm\frac{1}{\sqrt{2}}$ (Answers in all 4 quadrants, $\alpha = \frac{\pi}{4}$)

$x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

b) $\cos^2 x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x = 2$ — NO SOLN

$\cos x = -1$ — $x = \pi$

$-1 \leq \cos x \leq 1$

c) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = 1$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

d) $2\sec^2 x - 3 + \tan x = 0$

Recall: $\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$.

$2(\tan^2 x + 1) - 3 + \tan x = 0$

$2\tan^2 x + \tan x - 1 = 0$

$(2\tan x - 1)(\tan x + 1)$

$\tan x = \frac{1}{2}$ $\qquad$ $\tan x = -1$ $\quad \alpha = \pi/4$

$\alpha = 0.46$ $\qquad$ $\hookrightarrow x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = 0.46$ $\qquad$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \pi + 0.46$

$= 3.60$