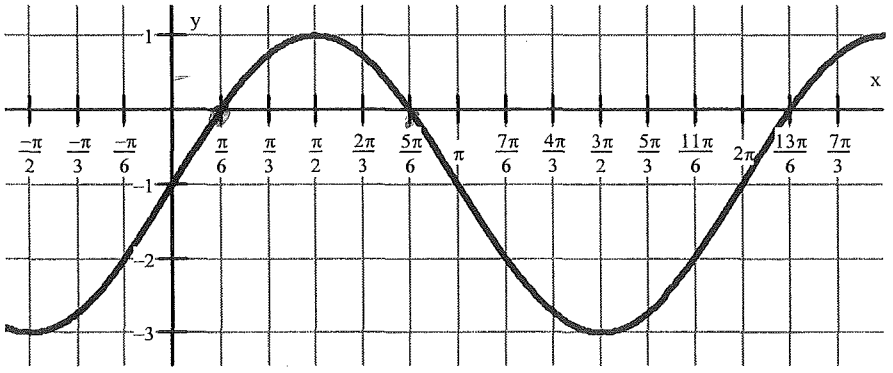
Day 6: 5.4 – Solving Linear Trig Equations

- **Solving** a trig equation means finding the values of x that make an equation true
- Because of their periodic nature, trigonometric equations have an infinite number of solutions. When we use a trigonometric model, we usually want solutions within a specified interval.
- We can solve trig equations **algebraically** or **graphically**

EX 1 – Solve $2\sin x - 1 = 0$ on the interval $[0, 2\pi]$ **graphically**

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}$ OR $\frac{5\pi}{6}$

EX 2 – Solve the given equations **algebraically on the interval $[0, 2\pi]$**

a. $\cos\theta = -\frac{\sqrt{3}}{2}$ $\alpha = \frac{\pi}{6}$

Q2: $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Q3: $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

b. $2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ $\alpha = \frac{\pi}{6}$

Q1: $x = \frac{\pi}{6}$

Q2: $x = \pi - \frac{\pi}{6}$

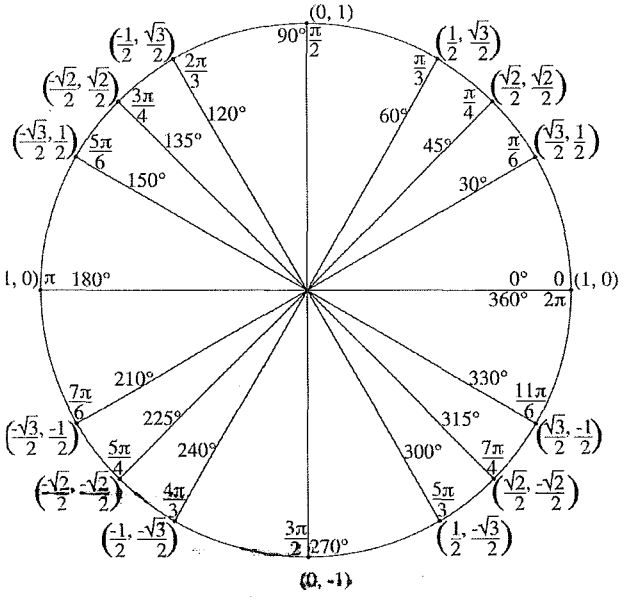
$= \frac{5\pi}{6}$

c. $2\sqrt{3}\tan x = -2$

$\tan x = \frac{-1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$

Q2: $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Q4: $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$



c. $3(\tan\theta - 1) = 2$ $\alpha = 1.03$

$\tan\theta - 1 = \frac{2}{3} \Rightarrow \tan\theta = \frac{5}{3}$

Q1: $\theta = 1.03$ rad

Q3: $\theta = \pi + 1.03$

$= 4.17$ rad

d. $2\csc\theta + 4 = 1 \Rightarrow \csc\theta = -\frac{3}{2}$

$\sin\theta = -\frac{2}{3}$ $\boxed{\alpha = 0.73}$

Q3: $\theta = \pi + 0.73 \doteq 3.87$ rad

Q4: $\theta = 2\pi - 0.73 = 5.55$

Practice: Determine the solutions for equation over the domain $[0, 2\pi]$. Round to two decimal places if necessary.

a) $\sin\theta = -\frac{\sqrt{3}}{2}$ $\alpha = \frac{\pi}{3}$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

b) $\tan\theta = 1$ $\alpha = \frac{\pi}{4}$

$\theta = \frac{\pi}{4}$

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

c) $\cos\theta = -\frac{1}{\sqrt{2}}$ $\alpha = \frac{\pi}{4}$

Q2: $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Q3: $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

d) $8\cot\theta = 9$

$\tan\theta = \frac{8}{9}$ $\alpha = 0.73$

Q1: $\theta = 0.73$ rad

Q3: $\theta = \pi + 0.73 = 3.87$ rad.

e) $2\sin x + 3 = 4$

$\sin x = 1/2$, $\alpha = \pi/6$

Q1: $x = \frac{\pi}{6}$

Q2: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

g) $3(\cot x + 2) = 4 \Rightarrow 3\cot x + 6 = 4 \Rightarrow \cot x = -\frac{2}{3}$

$\tan x = -\frac{3}{2}$ $\alpha = 0.98$

Q2: $x = \pi - 0.98 = 2.16$ rad

Q4: $x = 2\pi - 0.98 = 5.30$ rad

h) $2\cos x + \sqrt{3} = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2}$ $\alpha = \frac{\pi}{6}$

Q2: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Q3: $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

i) $4\sec\theta = 7 \Rightarrow \sec\theta = \frac{7}{4} \Rightarrow \cos\theta = \frac{4}{7}$

Q1: $\theta = 0.96$ rad.

Q4: $\theta = 2\pi - 0.96 \doteq 5.32$ rad

j) $3\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$

$\theta = \frac{\pi}{6}$

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

k) $-4\sin\theta - 2\sqrt{2} = 0 \Rightarrow +\sin\theta = \frac{+2\sqrt{2}}{-4} = -\frac{1}{\sqrt{2}}$

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$