Day 4/5: 5.3 – Transformations on Sinusoidal Functions

Recall from last year: Transformations of $y = \sin x$ and $y = \cos x$ are in the form:

$$y = \pm a\sin[\pm k(x-d)] + c \text{ and } y = \pm a\cos[\pm k(x-d)] + c$$

Functions can be graphed in two ways: using transformations or using properties

Parameter	Meaning when using transformations	Meaning when using properties
a	Vertical stretch ($a > 1$) by a factor of "a" Vertical compression ($0 < a < 1$) by a factor of "a" Reflection in the x-axis ($a < 0$)	Amplitude $= \lvert a \rvert$
k *Remember to factor!**	Horizontal stretch ($0 < k < 1$) by a factor of "$\frac{1}{k}$" Horizontal compression ($k > 1$) by a factor of "$\frac{1}{k}$" Reflection in the y-axis($k < 0$)	Period $= \frac{2\pi}{k}$ Or $k = \frac{2\pi}{period}$
d	Phase shift right ($d > 0$) "d" units Phase shift left ($d < 0$) "d" units	Phase shift
c	Vertical shift up ($c > 0$) "c" units Vertical shift down ($c < 0$) "c" units	Axis of the curve: $y = c$

- Recall that the k-value must be factored out from the d-value to get the correct phase shift:
- Example: $y = 3\sin(2x - \frac{\pi}{8}) + 1 \rightarrow y = 3\sin[2(x - \frac{\pi}{16})] + 1$,

  ∴ the phase shift is $\frac{\pi}{16}$ right.

EX 1 – Describe the transformations & state the key properties

$$y = 4\cos(2x - \frac{\pi}{3}) + 5 \rightarrow y = 4\cos[2(x - \frac{\pi}{6})] + 5$$

- Vertically stretched by a factor of 4
- Horizontally compressed by a factor of 1/2
- Horizontal translation $\frac{\pi}{6}$ ~~to~~ the right
- vertical translation 5 units upward

Amplitude	Period
4	$\frac{2\pi}{k} = \pi$
Phase Shift	**Axis of the Curve**
$\frac{\pi}{6}$ to the right	$y = 5$

To graph from properties:

1. Identify if the parent function *($y = \sin x$ or $y = \cos x$)*
2. Draw the axis of the curve *($y = c$)*
3. Indicate the amplitude ***(which gives the maximum and minimum values)***
4. Indicate the starting point ***(use phase shift)***
5. Indicate the ending point ***(based on the value of k)***
6. Indicate reflections ***(in the x-axis and/or y-axis)***

Then: sketch the number of indicated cycles or in the given domain

EX 2 – Graph the following functions using properties.

a) $y = 2\sin[\frac{1}{2}(x + \frac{\pi}{4})] - 1$ for **1 cycle**

$(x, y) \to (2x - \frac{\pi}{4}, 2y - 1)$

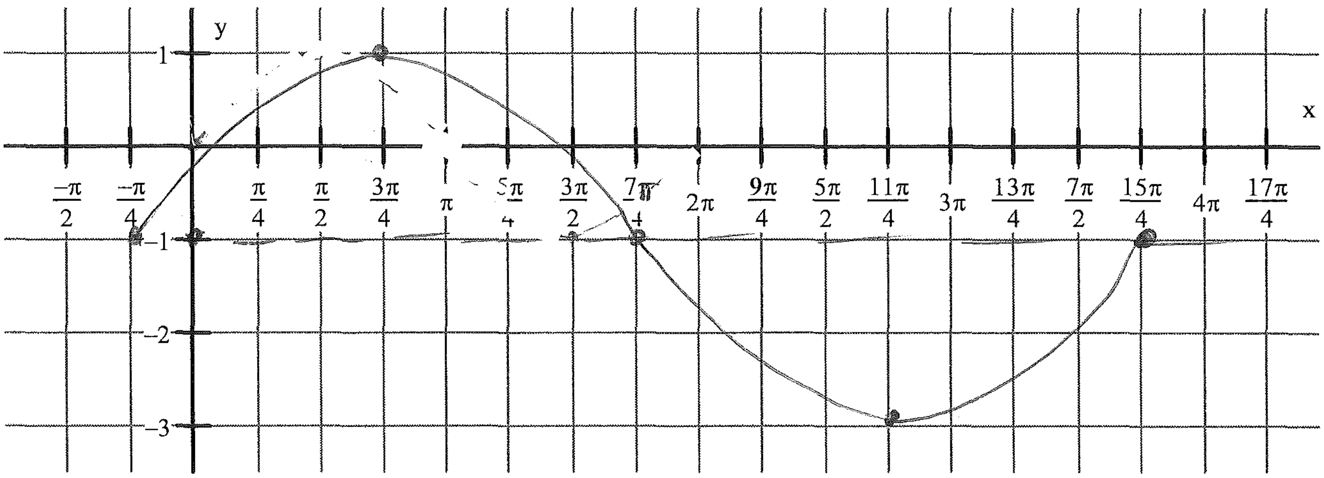
$(0, 0) \to (-\frac{\pi}{4}, -1)$

$(\frac{\pi}{2}, 1) \to (\frac{3\pi}{4}, 1)$

$(\pi, 0) \to (\frac{7\pi}{4}, -1)$

$(\frac{3\pi}{2}, -1)$ $(\frac{11\pi}{4}, -3)$

$(2\pi, 0) \to (\frac{15\pi}{4}, -1)$



max

$y = -1$

min

b) $y = -\cos(2x - \pi) + 1$ for **2 cycles**

$= -\cos(2(x - \frac{\pi}{2})) + 1$

$(x, y) \to (\frac{x}{2} + \frac{\pi}{2}, -y + 1)$

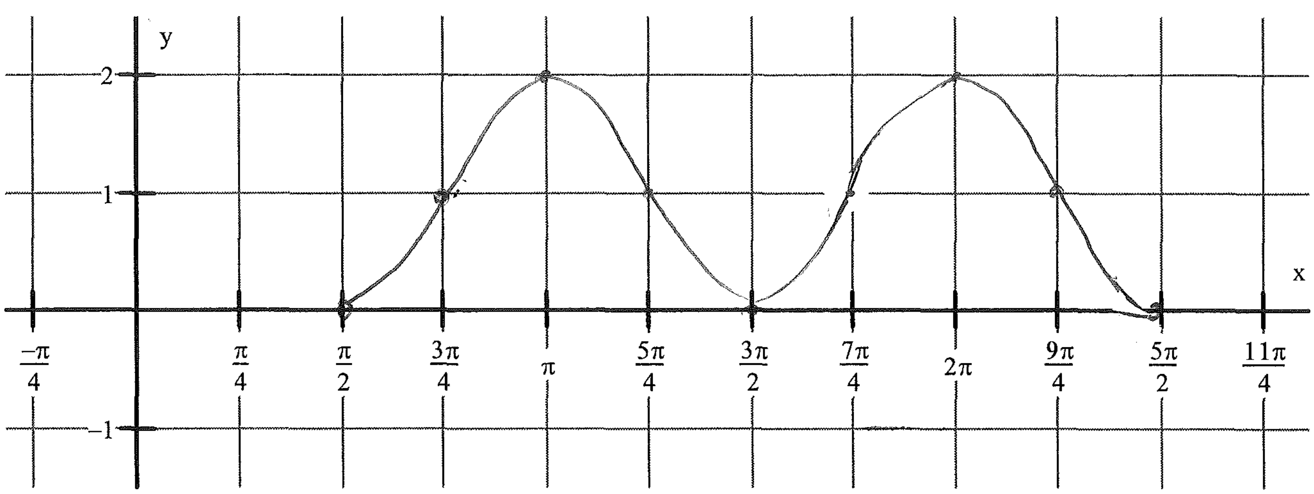
$(0, 1) \to (\frac{\pi}{2}, 0)$

$(\frac{\pi}{2}, 0) \to (\frac{3\pi}{4}, 1)$

$(\pi, -1) \to (\pi, 2)$

$(\frac{3\pi}{2}, 0) \to$

$(2\pi, 1) \to$

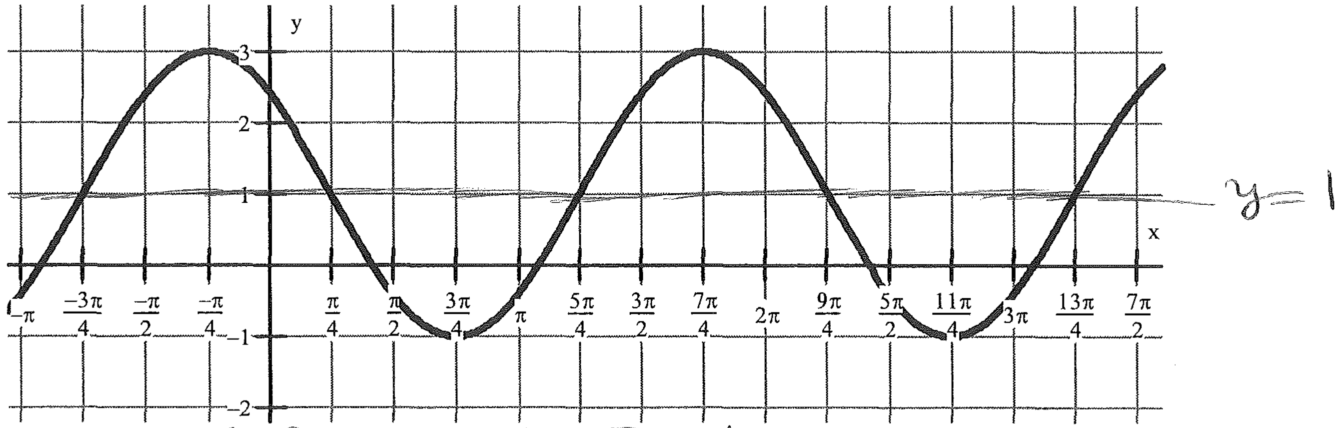


$y = 1$

> **To write an equation of a sinusoidal function:**
> - Identify the *amplitude*, *period* (then *k* value), *phase shift*, and *vertical shift*, and then substitute each into the appropriate place in the equation

EX 3 – Write the equation of each sinusoidal function given a graph for **both** sine and cosine

a)

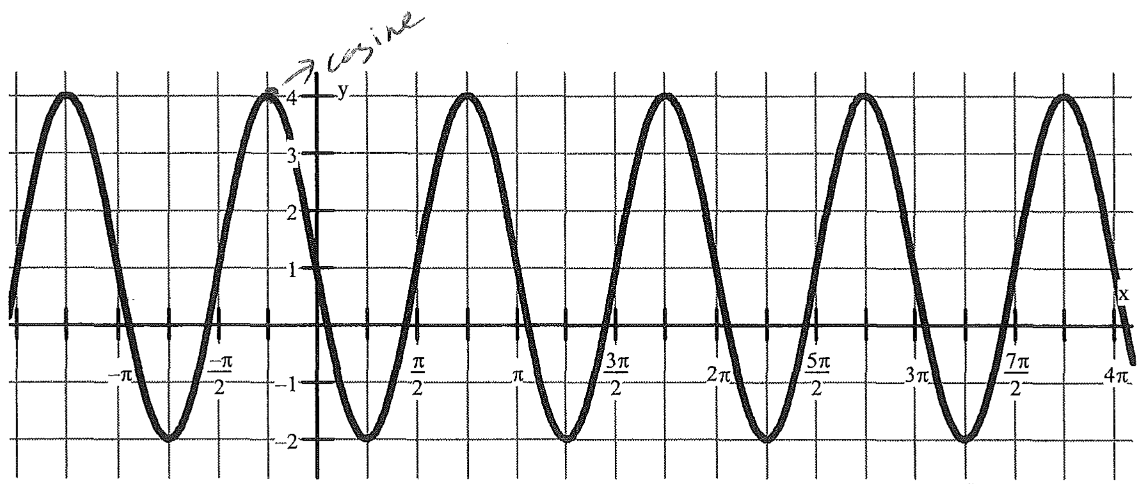


max $= 3$ } $a = 2$ Period $= 2\pi \Rightarrow k = 1$

min $= -1$ } $c = 1$

$y = 2\cos\left(x + \frac{\pi}{4}\right) + 1$ $y = 2\sin\left(x + \frac{3\pi}{4}\right) + 1$

b)



max $= 4$ } $a = 3$ Period $= \pi \Rightarrow k = 2$

min $= -2$ } $c = 1$

$y = -3\sin 2x + 1$ OR $y = 3\sin\left[2\left(x + \frac{\pi}{2}\right)\right] + 1$

$y = 3\cos\left[2\left(x + \frac{\pi}{4}\right)\right] + 1$