Unit 4 Pre-Test Review – Trig in Radians
MHF4U

Section 1: 4.1 – Radian Measure

1) Determine the approximate radian measure, to the nearest hundredth, for each angle.

a) 33°

$= \frac{33\pi}{180}$

$\doteq 0.58$ rad

b) 138°

$= \frac{138\pi}{180}$

$\doteq 2.41$ rad

c) 252°

$= \frac{252\pi}{180}$

$\doteq 4.40$ rad

d) 347°

$= \frac{347\pi}{180}$

$= 6.06$ rad

2) Determine the approximate degree measure, to the nearest tenth, for each angle.

a) 1.24

$= \frac{1.24(180)}{\pi}$

$\doteq 71.0°$

b) 2.82

$= 2.82\left(\frac{180}{\pi}\right)$

$\doteq 161.6°$

c) 4.78

$= \frac{4.78(180)}{\pi}$

$\doteq 273.9°$

d) 6.91

$= \frac{(6.91)(180)}{\pi}$

$\doteq 395.9°$

3) Determine the exact radian measure of each angle.

a) 75°

$= \frac{75\pi}{180}$

$= \frac{5\pi}{12}$ rad

b) 20°

$= \frac{20\pi}{180}$

$= \frac{\pi}{9}$ rad

c) 12°

$= \frac{12\pi}{180}$

$= \frac{\pi}{15}$ rad

d) 9°

$= \frac{9\pi}{180}$

$= \frac{\pi}{20}$ rad

4) Determine the exact degree measure of each angle.

a) $\frac{2\pi}{5}$

$= \frac{2(180)}{5}$

$= 72°$

b) $\frac{4\pi}{9}$

$= \frac{4(180)}{9}$

$= 80°$

c) $\frac{7\pi}{12}$

$= \frac{7(180)}{12}$

$= 105°$

d) $\frac{11\pi}{18}$

$= \frac{11(180)}{18}$

$= 110°$

5) An arc of a circle measuring 22.5 cm subtends a central angle of $\frac{4\pi}{3}$ radians. Find the approximate radius of the circle, to the nearest tenth of a cm.

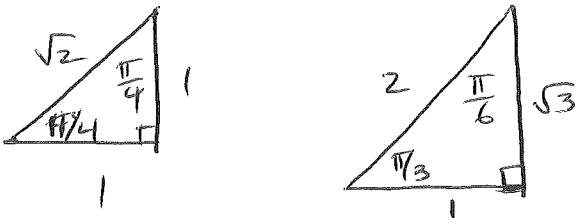
$a = r\theta$

$r = \frac{a}{\theta} = \frac{22.5}{4\pi/3}$

$\doteq 5.4$ cm

Section 2: 4.2 – Trig Ratios and Special Angles

6) Draw both special triangles using radian measures



7) Find the exact value (using CAST and drawing the special triangles) for …

a) $\sin\frac{5\pi}{3}$ → Q4

$= -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

b) $\cot\frac{\pi}{3}$

$= \frac{1}{\tan\pi/3} = \frac{1}{\sqrt{3}}$

c) $\cos\frac{5\pi}{4}$ ↓ Q3

$= -\cos\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

d) $\sec\frac{11\pi}{6} = \sec\frac{\pi}{6}$

$= \frac{1}{\cos\pi/6}$

$= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

e) $\tan 3\pi$ Point $(-1, 0)$

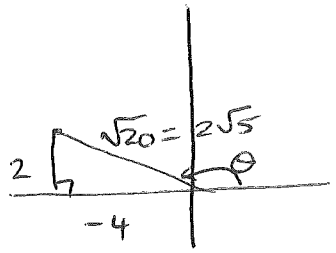
$= \tan\pi$

$= \frac{\sin\pi}{\cos\pi} = 0$

f) $\sin\frac{3\pi}{2}$ $(0, -1)$

$\sin\frac{3\pi}{2} = -1$

8) Suppose the terminal arm for the angle θ passes through the point (-4, 2). Find the exact values of $cot\theta$ and $sin\theta$.



$\cot\theta = \frac{A}{O}$

$= \frac{-4}{2}$

$= -2$

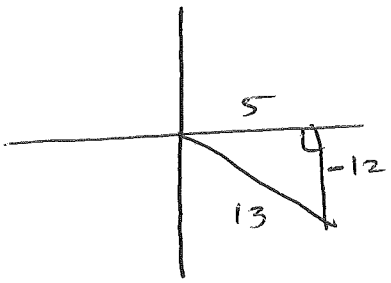
$\sin\theta = \frac{2}{2\sqrt{5}}$

$= \frac{1}{\sqrt{5}}$

$a^2 + b^2 = c^2$

$c = \sqrt{20} = 2\sqrt{5}$

9) Suppose $\sin\theta = -\frac{12}{13}$, and $\frac{3\pi}{2} \le \theta \le 2\pi$. Find $\sec\theta$ and $\tan\theta$.



$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{13}{5}$

$\tan\theta = \frac{-12}{5}$

$a^2 + b^2 = c^2 \Rightarrow a = 5$

10) Determine exact values for all 6 trig ratios of $\frac{5\pi}{3}$ radians.

A

$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\csc\frac{5\pi}{3} = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\sec\frac{5\pi}{3} = 2$

$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$

$\cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

10) Determine an exact value for each expression.

B

a) $\frac{\cot\frac{\pi}{4}}{\cos\frac{\pi}{3}\csc\frac{\pi}{2}}$

$= \frac{-1}{\left(\frac{1}{2}\right)(1)}$

$= 1 \times \frac{2}{1} = 2$

b) $\cos\frac{\pi}{6}\csc\frac{\pi}{3} + \sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\frac{2}{\sqrt{3}} + \frac{\sqrt{2}}{2}$

$= 1 + \frac{\sqrt{2}}{2}$

$= \frac{2+\sqrt{2}}{2}$

c) $\sec\left(\frac{5\pi}{4}\right) + \cot\left(\frac{2\pi}{3}\right)\sin\left(\frac{11\pi}{6}\right)$

$= -\sec\frac{\pi}{4} + \left[\cot\left(\frac{\pi}{3}\right)\right]\left[-\sin\frac{\pi}{6}\right]$

$= -\frac{1}{\cos\frac{\pi}{4}} + \frac{1}{\tan\frac{\pi}{3}}\sin\pi/6$

$= -\sqrt{2} + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{2}\right)$

$= -\sqrt{2} + \frac{1}{2\sqrt{3}} = \frac{-2\sqrt{6}+1}{2\sqrt{3}} = \frac{\sqrt{3}(2\sqrt{6}+1)}{6}$

Unit 5 Pre-Test Review – Trig Identities and Equations
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Section 1: Transformation and Co-function Identities

1 **1)** Given that $\sin\left(\frac{2\pi}{7}\right) \cong 0.7818$, use equivalent trigonometric expressions to evaluate the following,

a) $\cos\frac{3\pi}{14}$

$\frac{3\pi}{14} = \frac{\pi}{2} - a$

$a = \frac{\pi}{2} - \frac{3\pi}{14} = \frac{7\pi - 3\pi}{14} = \frac{4\pi}{14} = \frac{2\pi}{7}$

$\therefore \cos\left(\frac{3\pi}{14}\right) = \sin\left(\frac{2\pi}{7}\right) \doteq 0.7818$

b) $\cos\frac{11\pi}{14}$

$\frac{11\pi}{14} = \frac{\pi}{2} + a$

$a = \frac{11\pi}{14} - \frac{7\pi}{14} = \frac{4\pi}{14} = \frac{2\pi}{7}$

$\therefore \cos\left(\frac{11\pi}{14}\right) = -\sin\left(\frac{2\pi}{7}\right)$

$= -0.7818$

Section 2: Compound Angles

1 **2)** Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each.

a) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right)$

$= 1$

b) $\cos\frac{\pi}{3}\cos\frac{5\pi}{12} - \sin\frac{\pi}{3}\sin\frac{5\pi}{12}$ (a b a b)

$= \cos(a+b) = \cos\left(\frac{\pi}{3} + \frac{5\pi}{12}\right)$

$= \cos\left(\frac{4\pi + 5\pi}{12}\right) = \cos\left(\frac{9\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

1 **3)** Apply a compound angle formula, and then determine an exact value for each.

a) $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} + \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$= -\cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= -\frac{(\sqrt{6} + \sqrt{2})}{4}$

b) $\sin\left(\frac{5\pi}{4} - \frac{2\pi}{3}\right)$

$= \sin\frac{5\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{2\pi}{3}\cos\frac{5\pi}{4}$

$= \left(-\sin\frac{\pi}{4}\right)\left(-\cos\frac{\pi}{3}\right) - \left(\sin\frac{\pi}{3}\right)\left(-\sin\frac{\pi}{4}\right)$

$= +\frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$

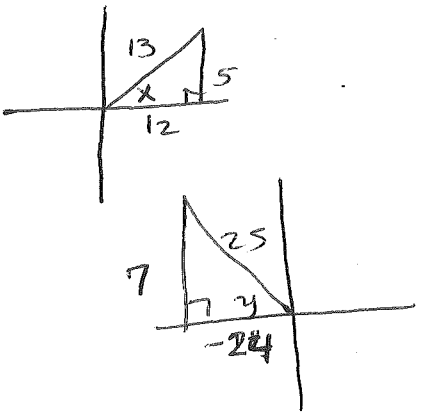
$= \frac{\sqrt{2} + \sqrt{6}}{4}$

14) Angle x is in the first quadrant and angle y is in the second quadrant such that $\cos x = \frac{12}{13}$ and $\sin y = \frac{7}{25}$. Determine an exact value for...

a) $\sin x = \frac{5}{13}$

b) $\cos y$

$= \frac{-24}{25}$



c) $\sin(x+y)$

$= \sin x\cos y + \sin y\cos x$

$= \left(\frac{5}{13}\right)\left(\frac{-24}{25}\right) + \left(\frac{7}{25}\right)\left(\frac{12}{13}\right)$

$= \frac{-120+84}{325} = \frac{-36}{325}$

d) $\sin(x-y)$

$= \sin x\cos y - \sin y\cos x$

$= \left(\frac{5}{13}\right)\left(\frac{-24}{25}\right) - \left(\frac{7}{25}\right)\left(\frac{12}{13}\right)$

$= \frac{-120-84}{325} = \frac{-204}{325}$

e) $\cos(x+y)$

$= \cos x\cos y - \sin x\sin y$

$= \left(\frac{12}{13}\right)\left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right)\left(\frac{7}{25}\right)$

$= \frac{-288-35}{325} = \frac{-323}{325}$

f) $\cos(x-y)$

$= \cos x\cos y + \sin x\sin y$

$= \left(\frac{12}{13}\right)\left(\frac{-24}{25}\right) + \left(\frac{5}{13}\right)\left(\frac{7}{25}\right)$

$= \frac{-288+35}{325} = \frac{-253}{325}$

15) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{11\pi}{12} = \sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

b) $\cos\frac{25\pi}{12}$ (Q1) $= \cos\left(\frac{25}{12}\pi - 2\pi\right) = \cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

$75° = 45° + 30°$

c) $\tan\frac{7\pi}{12} = -\tan\left(\pi - \frac{7\pi}{12}\right) = -\tan\left(\frac{5\pi}{12}\right) = -\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= -\left[\frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right] = -\left[\frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)}\right]$

$= -\left[\frac{3+\sqrt{3}}{3}\right] \div \left[\frac{3-\sqrt{3}}{3}\right] = -\frac{3+\sqrt{3}}{3} \cdot \frac{3}{3-\sqrt{3}}$

$= -\left[\frac{3+\sqrt{3}}{3-\sqrt{3}}\right]\cdot\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right) = -\left[\frac{9+6\sqrt{3}+3}{9-3}\right] = -\left[\frac{12+6\sqrt{3}}{6}\right] = -2-\sqrt{3}$

16) A 15-m ladder leaning against a wall is in an unsafe position if it makes an angle of less than $\frac{\pi}{12}$ radians with the wall. Use a compound angle formula to determine an exact expression for the minimum distance that the foot of the ladder can be placed from the wall so that the ladder is standing safely.

$\sin\frac{\pi}{12} = \frac{x}{15} \Rightarrow x = 15\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= 15\left[\sin\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{4}\right]$

$= 15\left[\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}\right]$

$= \frac{15}{4}(\sqrt{6}-\sqrt{2})$

Section 3: Double Angle Formulas

17) Express each of the following as a single trig ratio and then evaluate

a) $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

$= \sin\left(2\cdot\frac{\pi}{12}\right) = \sin\frac{\pi}{6}$

$= \frac{1}{2}$

b) $\cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)$

$= \cos\left(2\cdot\frac{\pi}{12}\right) = \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

c) $1 - 2\sin^2\left(\frac{3\pi}{8}\right)$

$= \cos\left(2\left(\frac{3\pi}{8}\right)\right)$

$= \cos\left(\frac{3\pi}{4}\right)$

$= -\cos\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

d) $\frac{2\tan\frac{\pi}{6}}{1-\tan^2\left(\frac{\pi}{6}\right)}$

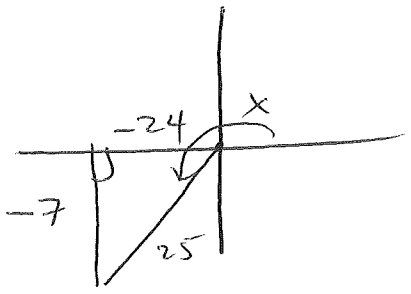
$= \tan\left(\frac{2\pi}{6}\right) = \tan\left(\frac{\pi}{3}\right)$

$= \tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

18) Angle x lies in the third quadrant, and $\tan x = \frac{7}{24}$.

a) Determine an exact value for $\cos(2x)$



$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{-24}{25}\right)^2 - \left(\frac{-7}{25}\right)^2$

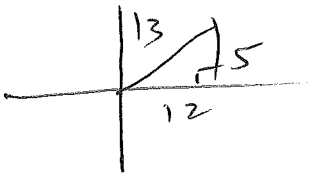
$= \frac{24^2 - 7^2}{625} = \frac{527}{625}$

b) Determine an exact value for $\sin(2x)$

$= 2\sin x\cos x = 2\left(\frac{-7}{25}\right)\left(\frac{-24}{25}\right)$

$= \frac{336}{625}$

19) Given $\sin x = \frac{5}{13}$ and $0 \le x \le \frac{\pi}{2}$, find $\sin(2x)$



$\sin 2x = 2\sin x\cos x$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$

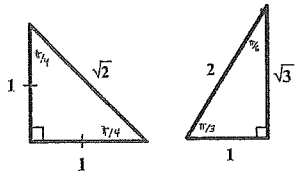
$= \frac{120}{169}$

Answer Key

1)a) 0.58 b) 2.41 c) 4.4 d) 6.06 2)a) 71° b) 161.6° c) 273.9 d) 395.9°

3)a) $\frac{5\pi}{12}$ b) $\frac{\pi}{9}$ c) $\frac{\pi}{15}$ d) $\frac{\pi}{20}$ 4)a) 72° b) 80° c) 105° d) 110° 5) 5.4 cm

6)



7)a) $-\frac{\sqrt{3}}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $-\frac{1}{\sqrt{2}}$ d) $\frac{2}{\sqrt{3}}$ e) 0 f) -1

8) $cot\theta = -2 \quad and \quad sin\theta = \frac{1}{\sqrt{5}}$

9) $\sec\theta = \frac{13}{5}, \tan\theta = -\frac{12}{5}$

10 A) $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$; $\cos\frac{5\pi}{3} = \frac{1}{2}$; $\tan\frac{5\pi}{3} = -\sqrt{3}$; $\csc\frac{5\pi}{3} = \frac{-2}{\sqrt{3}}$; $\sec\frac{5\pi}{3} = 2$; $\cot\frac{5\pi}{3} = \frac{-1}{\sqrt{3}}$

10 B)a) 2 b) $\frac{\sqrt{2}+1}{\sqrt{2}}$ c) $\frac{-2\sqrt{6}+1}{2\sqrt{3}}$

ANSWER KEY

11)a) 0.7818 b) -0.7818

12)a) $\sin\frac{\pi}{2} = 1$ b) $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

13)a) $\frac{-\sqrt{3}+1}{2\sqrt{2}}$ b) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

14)a) $\frac{5}{13}$ b) $-\frac{24}{25}$ c) $-\frac{36}{325}$ d) $-\frac{204}{325}$ e) $-\frac{323}{325}$ f) $-\frac{253}{325}$

15)a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ b) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ c) $-2-\sqrt{3}$

16) $15\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ meters

17)a) $\sin\frac{\pi}{6} = \frac{1}{2}$ b) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ c) $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ d) $\tan\frac{\pi}{3} = \sqrt{3}$

18) a) $\frac{527}{625}$ b) $\frac{336}{625}$

19) $\frac{120}{169}$