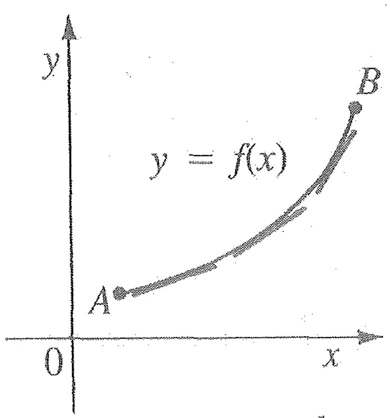
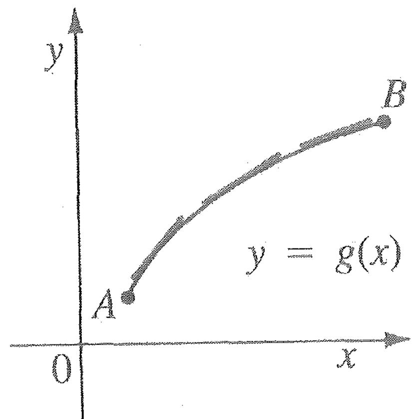
Day 4: 4.4 Concavities and Points of Inflection

INVESTIGATION:

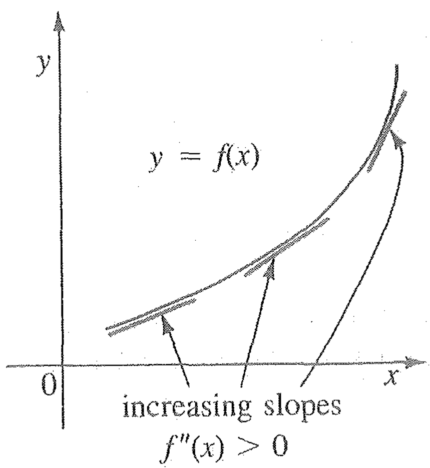


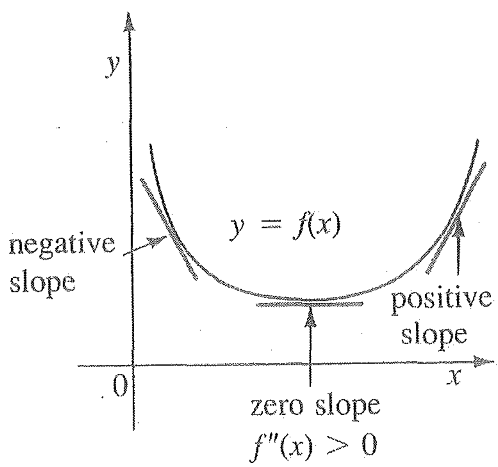
concave upward



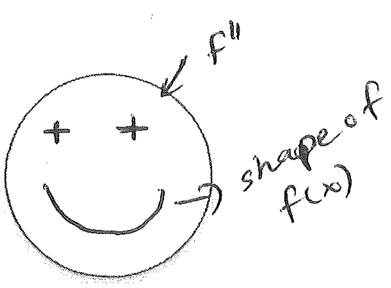
concave downward

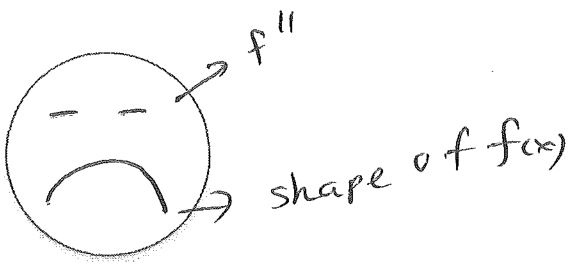
What do we notice?

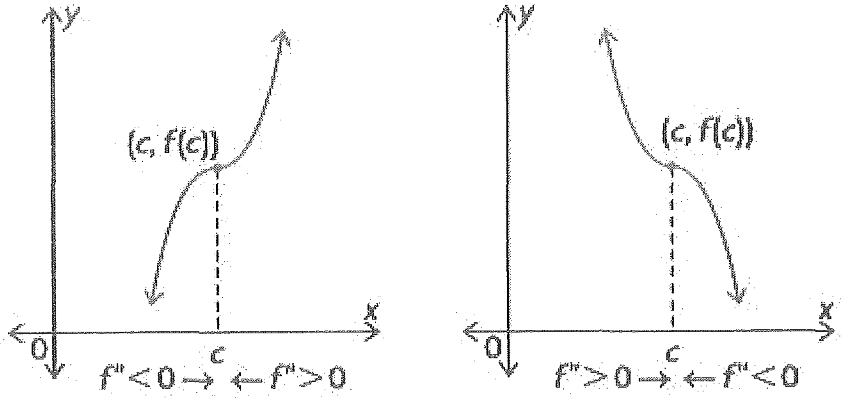




What do we notice?



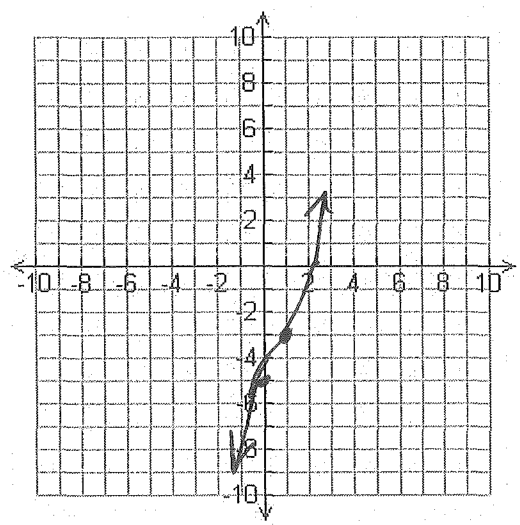




What do we notice?

at $x=c$, there is a point of inflection
[point where the concavity changes].

Example 1
(a) Determine where the curve $y = x^3 - 3x^2 + 4x - 5$ is concave upward and where it is concave downward.
(b) Find the points of inflection.
(c) Use this information to sketch the curve.



$y' = 3x^2 - 6x + 4$

$y'' = 6x - 6$

$6x - 6 = 0 \Rightarrow x = 1$

$x = 1$ is a potential POI

we need to check f'' changes sign at $x = 1$

x	$(-\infty, 1)$	$(1, \infty)$
$f''(x)$	$-$	$+$
$f(x)$	$\frown$	$\smile$

$\therefore f(x)$ is CU when $x \in (1, \infty)$
CD when $x \in (-\infty, 1)$

at $(1, -3)$ there is a point of inflection.

Example 2: $y = x^3(x-8)$

a) Determine where the curve $y = x^4 - 8x^3$ is concave upward and where it is concave downward.

b) Find the points of inflection.

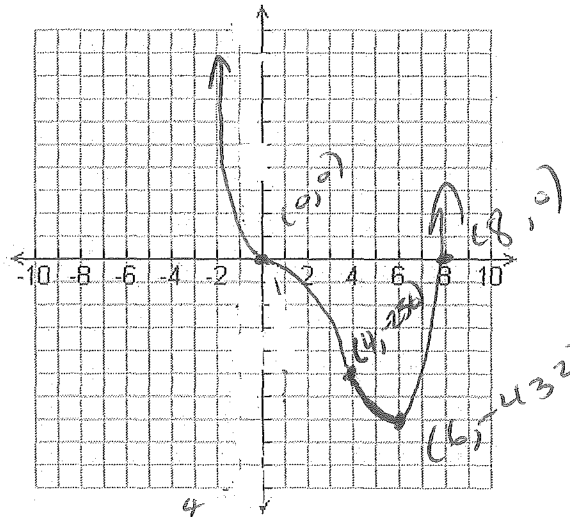
c) Use this information to sketch the curve.

$y' = 4x^3 - 24x^2 = 4x(x-6)$

$y'' = 12x^2 - 48x$

$= 12x(x-4)$

$\therefore x = 0, 4$ potential x-values of POI.



x	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
$f''(x)$	+	−	+
$f(x)$	CU ∪	CD ∩	CU ∪

$(0,0)$ POI and $(4,-256)$ POI. $(6,-432)$ local min.

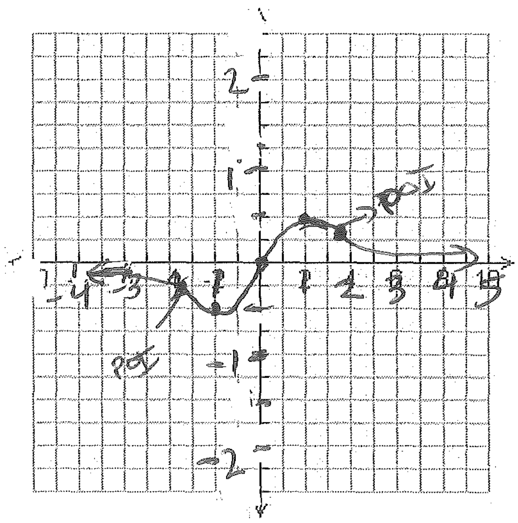
Example 3: Discuss the $y = \dfrac{x}{x^2+1}$ curve with respect to concavity and points of inflection if it is given that

$$y' = \frac{1-x^2}{(x^2+1)^2} \quad \text{and} \quad y'' = \frac{2x(x^2-3)}{(x^2+1)^3}$$

$y'' = 0 \Rightarrow 2x(x^2-3) = 0$

$x = 0$ or $x^2 = 3$

$x = \pm\sqrt{3}$



x	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(0, \sqrt{3})$	$(\sqrt{3}, \infty)$
$f''(x)$	−	+	−	+
$f(x)$	CD ∩	CU ∪	CD ∩	CU ∪

Points of inflection $\left\{ \begin{array}{l} \left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right) \\ (0, 0) \\ \left(\sqrt{3}, \frac{\sqrt{3}}{4}\right) \end{array} \right.$

$(-1.73, -0.43)$

$(1.73, 0.43)$

NOTE: $y' = 0$ when $x = 1, -1$

y': − (−1) + (1) −

$\therefore (-1, \frac{-1}{2})$ local min

$(1, \frac{1}{2})$ local max