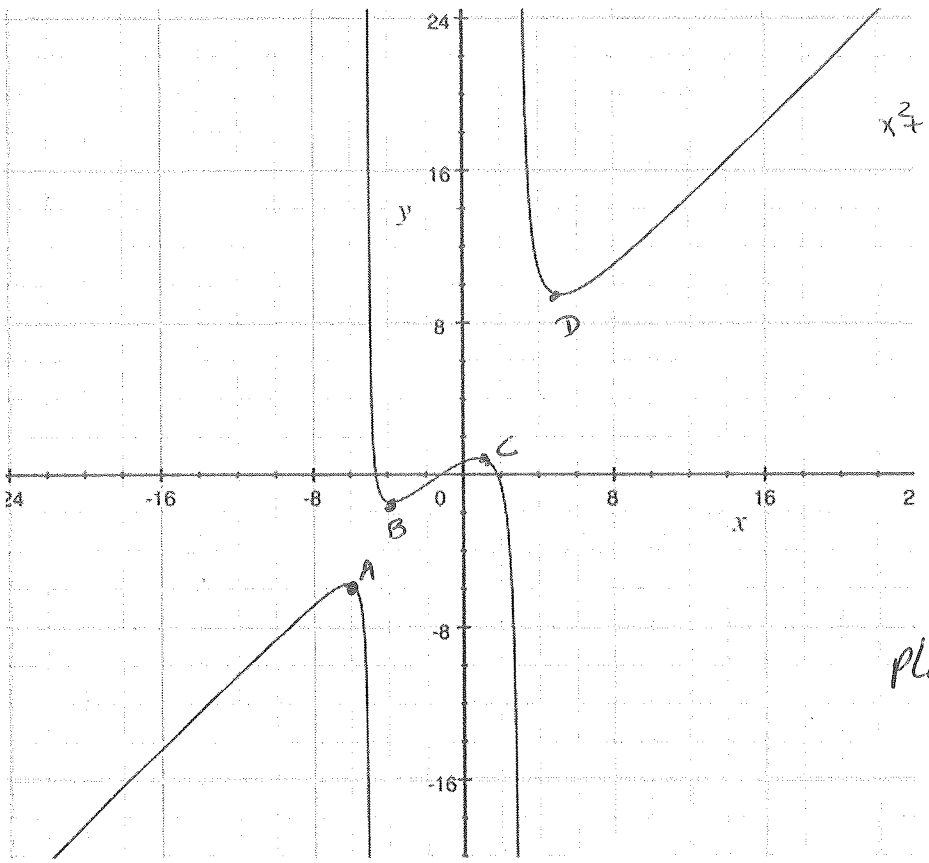
Day 1: 4.1 Increasing and Decreasing Functions

INVESTIGATION:

The graph of the function $y = \frac{x^3 + 4x^2 - 5x - 10}{x^2 + 2x - 15} = \frac{x^3 + 4x^2 - 5x - 10}{(x+5)(x-3)}$



$$\begin{array}{r} x+2 \\ x^2+2x-15 \overline{) x^3+4x^2-5x-10} \\ x^3+2x^2-15x \\ \hline 2x^2+10x-10 \\ 2x^2+4x-30 \\ \hline 6x+20 \end{array}$$

$\therefore y = x+2$ is the oblique asymptote.

Please NOTE: $\left.\begin{array}{l} x=-5 \\ x=3 \end{array}\right\}$ VA

NOT PART OF THE DOMAIN.

Discussion: IMPORTANT INFORMATION

Domain: $\{x \in \mathbb{R} \mid x \neq -5, 3\}$

Range $\{y \in \mathbb{R}\}$

Asymptote(s): VA: $x = -5, 3$

HA: None

oblique: $y = x+2$

End Behaviour: $x \to \infty \quad y \to (x+2)^+$

$x \to -\infty \quad y \to (x+2)^-$

Special Points

Intervals:

- *Intercepts* x-int: $-4.5, 1.9, -0.9$

  y-int: $\frac{-10}{-15} = \frac{2}{3}$

- *Local (Relative) Maxima/Minima*

  Local max: $A(-6,-6)$ $C(1.2, 0.7)$

  min $B(-4,-1.9)$ $D(5, 9.8)$

- *Inflection*

  $(-0.9, 0)$

Increase: $x \in (-\infty, -6) \cup (-4, 1.2) \cup (5, \infty)$

Decrease:

$x \in (-6, -5) \cup (-5, -4) \cup (1.2, 3)$

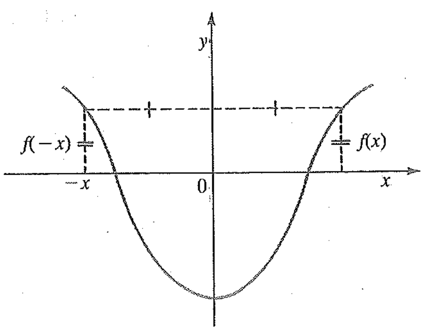
$\cup (3, 5)$

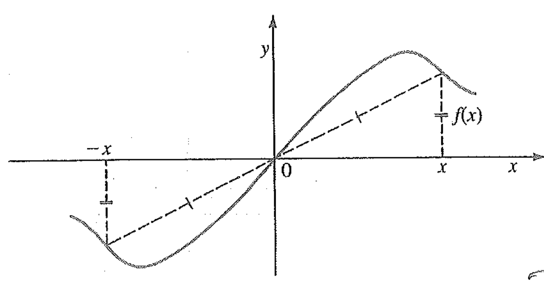
INTERVAL NOTATIONS

Remember this table?

Bracket Interval	Inequality	Number Line	In Words
			The set of all real numbers x such that
(a, b)	$a < x < b$	a b $\mathbb{R}$	x is greater than a and less than b
$(a, b]$	$a < x \le b$	a b $\mathbb{R}$	x is greater than a and less than or equal to b
$[a, b)$	$a \le x < b$	a b $\mathbb{R}$	x is greater than or equal to a and less than b
$[a, b]$	$a \le x \le b$	a b $\mathbb{R}$	x is greater than or equal to a and less than or equal to b
$[a, \infty)$	$x \ge a$	a $\mathbb{R}$	x is greater than or equal to a
$(-\infty, a]$	$x \le a$	a $\mathbb{R}$	x is less than or equal to a
(a, ∞)	$x > a$	a $\mathbb{R}$	x is greater than a
$(-\infty, a)$	$x < a$	a $\mathbb{R}$	x is less than a
$(-\infty, \infty)$	$-\infty < x < \infty$	$\mathbb{R}$	x is an element of the real numbers

EVEN OR ODD FUNCTION?





$f(x) = f(-x)$

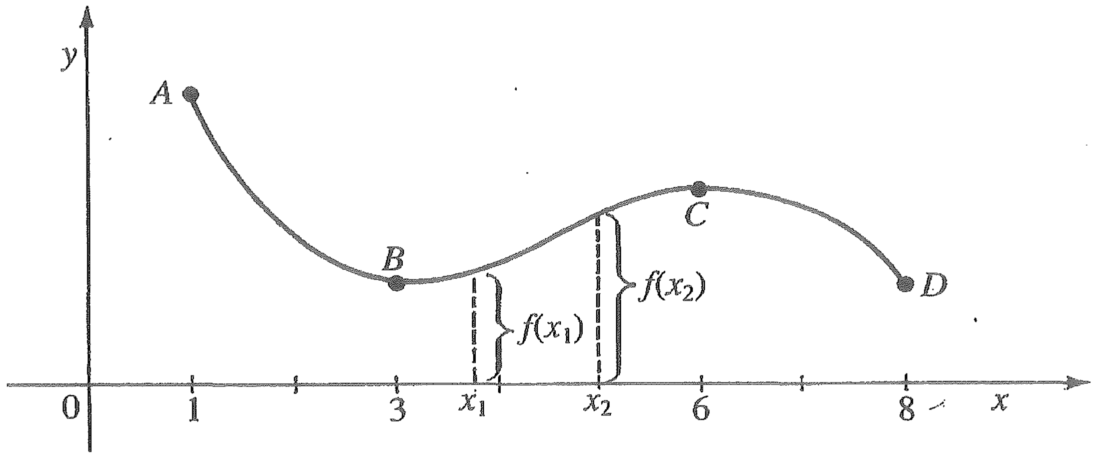
Then $f(x)$ is an even function

$f(x) = -f(-x)$

OR

$-f(x) = f(-x)$

ODD FUNCTION



In general, a function f is called **increasing on an interval I** if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in I}$$

It is called **decreasing on I** if

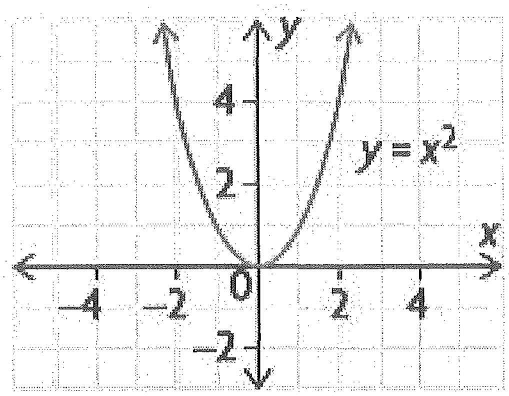
$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in I}$$

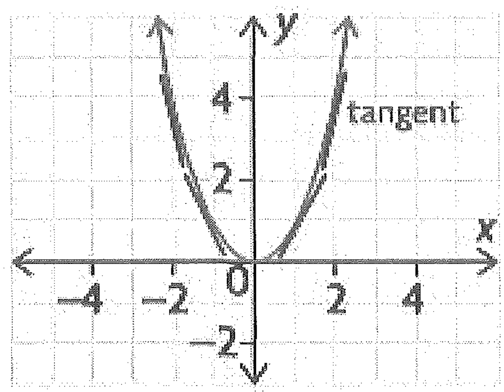
Which interval the above function is increasing? Which interval it is decreasing?

inc: $x \in (3, 6)$

dec: $x \in (1, 3) \cup (6, 8)$

State the interval that y = x^2 is increasing; the interval it is decreasing. $x \in (-\infty, 0)$





What do you notice about the tangent line in each interval?

Tangents have a negative slope if the f(x) is decreasing.

$x \in (-\infty, 0)$ f(x) is decreasing

$x \in (0, \infty)$ f(x) is increasing

NOTE! @ $x = 0$ the function is neither inc / dec.

<u>Test for Increasing & Decreasing Intervals</u>

1. *If $f'(x) > 0$ for all x in an interval I, then f(x) is increasing on I*
2. *If $f'(x) < 0$ for all x in I, then f(x) is decreasing on I*

Example 1: Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing

set $f' = 0$

$8x - 5 = 0$

$x = \frac{5}{8}$

$\frac{5}{8}$

x	$(-\infty, 5/8)$	$(5/8, \infty)$
$f'(x)$	$-$	$+$
$f(x)$	decreasing	increasing

dec → inc implies at $x = \frac{5}{8}$ local min

Example 2 Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

$y' = 0 \Rightarrow 3x^2 + 12x + 9 = 0$

$3(x^2 + 4x + 3) = 0$

$3(x+3)(x+1) = 0$

$x = -3, -1$

$-3 \quad -1$

3 intervals.

x	$(-\infty, -3)$	$(-3, -1)$	$(-1, \infty)$
$3(x+3)$	$-$	$+$	$+$
$(x+1)$	$-$	$-$	$+$
$f'(x)$	$+$	$-$	$+$
$f(x)$	increasing	decreasing	increasing

$(-3, 2)$ local max $\quad (-1, -2)$ local min

Example 3 Find the intervals of increase and decrease for the function $g(x) = x^4 - 4x^3 - 8x^2 - 1$

$4x^3 - 12x^2 - 16x = 0$

$4x(x^2 - 3x - 4) = 0$

$4x(x-4)(x+1) = 0$

$-1 \quad 0 \quad 4$

x	$(-\infty, -1)$	$(-1, 0)$	$(0, 4)$	$(4, \infty)$
$(4x)$	$-$	$-$	$+$	$+$
$(x-4)$	$-$	$-$	$-$	$+$
$(x+1)$	$-$	$+$	$+$	$+$
$f'(x)$	$-$	$+$	$-$	$+$
$f(x)$	dec	inc	dec	inc.

$\therefore$ f(x) is dec if $x \in (-\infty, -1) \cup (0, 4)$

f(x) is inc if $x \in (-1, 0) \cup (4, \infty)$

$(-1, -12) \quad (4, -129)$ LOCAL MIN

$(0, -1)$ LOCAL MAX