Day 6: 3.4 Optimization in Science & Economy

Cost/Revenue/Profit Problem

- *Let x is quantity of products sale/produced*
- *Cost as a function of quantity of products C(x)*
- *Revenue as a function of quantity of products R(x)*
- *Profit as a function of quantity of products P(x) = R(x) – C(x)*
- *Average cost (unit cost) =* $\frac{C(x)}{x}$

Objective: Minimize the cost or maximize the Profit or the Revenue.

Example 1: For an outdoor concert, a ticket of \$30 normally attracts 5000 people. For each \$1 increase in the ticket price, 100 **fewer** people will attend. What ticket price will **maximize the revenue**?

Let x represent number of price increases.

$R(x) = (30 + x)(5000 - 100x)$

$R'(x) = 0 \Rightarrow 1(5000 - 100x) + (30 + x)(-100) = 0$

$5000 - 100x - 3000 - 100x = 0$

$2000 - 200x = 0$

$x = 2$

$\therefore P(x) = 30 + x$

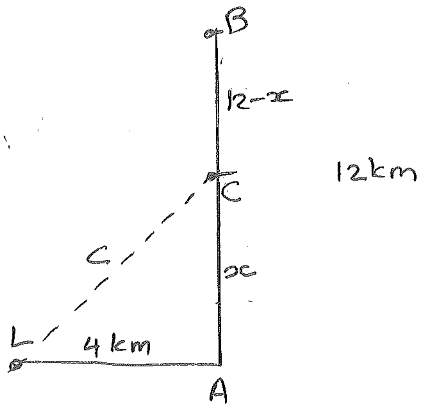
$P(2) = 30 + 2$

$= \$32$

$\therefore$ Ticket price of \$32 would maximize the revenue.

NOTE: This concept was covered in gr 10 and 11. We can complete the square to find the max/min.

Example 2: A lighthouse, L, is located on a small island 4km East of point A on a straight North-South coastline. A power cable is to be laid from to L to the nearest power station at point B on the shoreline 12km North of A. The cost of laying cable under the water is \$6000/km and the cost of laying cable along the shoreline is \$2000/km. Find the location of point C between A and B on the shoreline where the power cable should enter the water to minimize the cost?



$c^2 = 4^2 + x^2$

$c = \sqrt{16+x^2}$

$0 \leq x \leq 12$

$$C(x) = 6000\sqrt{16+x^2} + 2000(12-x)$$

$$C'(x) = 6000\left(\frac{1}{\cancel{2}\sqrt{16+x^2}}\right)(\cancel{2}x) - 2000$$

$$C'(x) = 0 \Rightarrow \quad 2000 = \frac{6000x}{\sqrt{16+x^2}}$$

$$2000\sqrt{16+x^2} = 6000x$$

$$\sqrt{16+x^2} = 3x$$

$$16 + x^2 = 9x^2$$

$$8x^2 = 16$$

$$x^2 = 2$$

$$x = \sqrt{2} \approx 1.41 \text{ km.}$$

$\therefore$ The point C should be 1.41 km North of point A.