

Day 2: Polynomial Models Involving Velocity and Speed

In many situations, an object's position, s , can be represented as $s(t)$. Recall that

Average velocity is defined as the rate of change of displacement over an interval -of time.

Instantaneous velocity is the rate of change of displacement at a specific point in time.

On a displacement-time graph, the slope of a secant represents average velocity, while the slope of a tangent represents instantaneous velocity.

The rate of change of $s(t)$ with respect to time is the object's **velocity**, $v(t)$, and the rate of change of the velocity with respect to time is its **acceleration**, $a(t)$. The absolute value/magnitude of the velocity is called **speed**.

$s(t)$	distance function
$s'(t) = v(t)$	velocity function
$s''(t) = v'(t) = a(t)$	acceleration function

In Leibniz notation: $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$s(t)$, $v(t)$, $a(t)$ could be positive or negative depends on the direction of the object

The speed is $|v(t)|$, **always positive**, regardless of the direction

Key Concepts:

- Positive velocity means object is moving in positive direction
- Negative velocity means object is moving in negative direction.
- Object is at rest means object is not moving which implies velocity = 0
- Negative acceleration means velocity is decreasing.
- Positive acceleration means velocity is increasing.
- Zero acceleration means $a(t)=0$ which means velocity is constant which means object is neither accelerating or decelerating.
- An object is accelerating (speeding up) when its velocity and acceleration have the same signs.
- An object is decelerating (slowing down) when its velocity and acceleration have opposite signs.

Example 1: The position function of a moving object is $s(t) = t^3 - 15t^2 + 63t$ $t \geq 0$ where s is in metres, t is in seconds.

a) Find the velocity $v(t)$ and acceleration function $a(t)$.

$$v(t) = s'(t) = 3t^2 - 30t + 63, \quad t \geq 0$$

$$a(t) = v'(t) = s''(t) = 6t - 30, \quad t \geq 0$$

b) Find the average velocity during the time interval of [1, 3].

$$AVOC = \frac{s(3) - s(1)}{3 - 1}$$

$$s(3) = 81$$

$$s(1) = 49$$

$$= \frac{81 - 49}{2}$$

$$= 16 \text{ m/s}$$

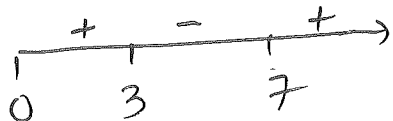
c) Find the time instant at which the object is at rest. $\Rightarrow v(t) = 0$

$$3t^2 - 30t + 63 = 0$$

$$3(t^2 - 10t + 21) = 0$$

$$\Rightarrow (t-3)(t-7) = 0 \Rightarrow t = 3 \text{ sec and } t = 7 \text{ sec}$$

d) When is the particle moving in a positive direction. $v(t) > 0$



$$\therefore (3, f(3)) \text{ max}$$

$$(7, f(7)) \text{ min.}$$

$$\therefore t \in [0, 3) \cup (7, \infty)$$

$$0 \leq t < 3 \text{ or } t > 7$$

Time (s)	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
s(t)	0	49	74	81	76	65	54	49	56
v(t)	63	36	15	0	-9	-12	-9	0	15
a(t)	-30	-24	-18	-12	-6	0	6	12	18

e) Fill the following table

f) Draw a diagram to show the movement of the object during the first 10 s ?

