Day 2: Polynomial Models Involving Velocity and Speed

In many situations, an object's position, s, can be represented as s(t). Recall that
Average velocity is defined as the rate of change of displacement over an interval -of time.
Instantaneous velocity is the rate of change of displacement at a specific point in time.
On a displacement-time graph, the slope of a secant represents average velocity, while the slope of a tangent represents instantaneous velocity.

The rate of change of $s(t)$ with respect to time is the object's **velocity**, $v(t)$, and the rate of change of the velocity with respect to time is its **acceleration**, $a(t)$. The <u>absolute value/magnitude</u> of the velocity is called **speed**.

$s(t)$	distance function
$s'(t) = v(t)$	velocity function
$s''(t) = v'(t) = a(t)$	acceleration function

In Leibniz notation: $v = \frac{ds}{dy}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

s(t), v(t), a(t) could be positive or negative depends on the direction of the object

The speed is $|v(t)|$, **<u>always positive</u>**, regardless of the direction

Key Concepts:

- *Positive velocity means object is moving in* positive direction
- *Negative velocity means object is moving in* negative direction.
- *Object is at rest means object is not moving which implies* velocity = 0
- *Negative acceleration means velocity is* decreasing.
- *Positive acceleration means velocity is* increasing.
- *Zero acceleration means a(t)=0 which means velocity is constant which means object is neither accelerating or decelerating.*
- *An object is accelerating (speeding up) when its velocity and acceleration have the same signs.*
- *An object is decelerating (slowing down) when its velocity and acceleration have opposite signs.*

Example 1: The position function of a moving object is $s(t) = t^3 - 15t^2 + 63t \quad t \geq 0$ where s is in metres, t is in seconds.

a) Find the velocity v(t) and acceleration function a(t).

$v(t) = s'(t) = 3t^2 - 30t + 63, \quad t \geq 0$

$a(t) = v'(t) = s''(t) = 6t - 30, \quad t \geq 0$

b) Find the average velocity during the time interval of [1, 3].

$AROC = \dfrac{s(3)-s(1)}{3-1}$ $\qquad s(3) = 81$

$s(1) = 49$

$= \dfrac{81-49}{2}$

$= 16 \text{ m/s}$

c) Find the time instant at which the object is at rest. $\Rightarrow v(t) = 0$

$3t^2 - 30t + 63 = 0$

$3(t^2 - 10t + 21) = 0$

$3(t-3)(t-7) = 0 \Rightarrow t = 3 \text{ sec}$ and $t = 7 \text{ sec}$

d) When is the particle moving in a positive direction. $v(t) > 0$

$+ \quad - \quad +$

$0 \quad 3 \quad 7$

$\therefore (3, f(3))$ max

$(7, f(7))$ min.

$\therefore \quad t \in [0,3) \cup (7, \infty)$

$0 \leq t < 3$ or $t > 7$

Time (s)	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8
s(t)	0	49	74	81	76	65	54	49	56
v(t)	63	36	15	0	−9	−12	−9	0	15
a(t)	−30	−24	−18	−12	−6	0	6	12	18

e) Fill the following table

f) Draw a diagram to show the movement of the object during the first 10 s ?