Day 1: 3.1 High-Order Derivatives

HIGH ORDER DERIVATIVES:

If the function $y = f(x)$ *is differentiable then* $y' = f'(x)$ is the **first** derivative of y = f(x) w.r.t. x. If y' = f'(x) is also differentiable then y'' = f''(x) is the **second derivative** of y = f(x) w.r.t. x

Notations:

$$y'' = f''(x) = \frac{d^2y}{dx^2} = \frac{d^2}{dx^2}f(x)$$

$$y''' = f'''(x) = \frac{d^3y}{dx^3} = \frac{d^3}{dx^3}f(x)$$

$$y^{(4)} = f^{(4)}(x) = \frac{d^4y}{dx^4} = \frac{d^4}{dx^4}f(x)$$

$$y^{(n)} = f^{(n)}(x) = \frac{d^ny}{dx^n} = \frac{d^n}{dx^n}f(x)$$

Warm-up: Find the derivative of $f(x) = x^4 + 5x^3 - 6x + 7$. Then find the derivative of the derivative. Repeat until you are left with a constant.

$f'(x) = 4x^3 + 15x^2 - 6$

$f''(x) = 12x^2 + 30x$

$f'''(x) = 24x + 30$

$f^4(x) = 24$

Note: degree of $f(x)$ is 4.
$f^4(x)$ is constant.

Example 1:

Function, $f(x)$	First Derivative, $f'(x)$	Second Derivative, $f''(x)$
a) $f(x)=3x^2+7x-5$	$f'(x)=6x+7$	$f''(x)=6$
b) $f(x)=(2x+3)^2$	$f'(x)=2(2x+3)^1(2)$ $=4(2x+3)=8x+12$	$f''(x)=8$
c) $f(x)=3x^4-5x^3+4x^2-8x+3$	$f'(x)=12x^3-15x^2+8x-8$	$f''(x)=36x^2-30x+8$
d) $f(x)=\dfrac{x+3}{x-5}$, $x \neq 5$	$f'(x)=\dfrac{1(x-5)-1(x+3)}{(x-5)^2}$ $=\dfrac{x-5-x-3}{(x-5)^2}$ $=\dfrac{-8}{(x-5)^2}$	$f'(x)=-8(x-5)^{-2}$ $f''(x)=16(x-5)^{-3}$ $=\dfrac{16}{(x-5)^3}$
e) $f(x)=\dfrac{x}{x^2-16}$, $x \neq \pm 4$	$f'(x)=\dfrac{1(x^2-16)-(x)(2x)}{(x^2-16)^2}$ $=\dfrac{x^2-16-2x^2}{(x^2-16)^2}$ $=\dfrac{-x^2-16}{(x^2-16)^2}$	$f''=\dfrac{-2x(x^2-16)^2-(-x^2-16)(2)(x^2-16)(2x)}{(x^2-16)^4}$ $=\dfrac{-2x(x^2-16)[x^2-16-2x^2-32]}{(x^2-16)^4}$ $=\dfrac{-2x[-x^2-48]}{(x^2-16)^3}$ $=\dfrac{2x(x^2+48)}{(x^2-16)^3}$