Unit Review – Line Segments

1. Given the following points: Y(6, -1) and O(-3, 5), find:

a. Length of YO

$\overline{YO} = \sqrt{(-3-6)^2 + [5-(-1)]^2}$

$= \sqrt{81 + 36}$ either correct

$= \boxed{\sqrt{117}} = \underbrace{\sqrt{9 \cdot 13} = \sqrt{9}\cdot\sqrt{13}}_{\text{simplification}} = \boxed{3\sqrt{3}}$

b. Midpoint of YO

$M_{YO} = \left(\frac{-3+6}{2}, \frac{5+(-1)}{2}\right)$

$= (1.5, 2)$

$M(1.5, 2)$

c. Slope of YO

$m_{YO} = \frac{5-(-1)}{-3-6} = \frac{5+1}{-9} = -\frac{6}{9} = -\frac{2}{3}$

d. Equation of the line YO

$m = -2/3 \quad Y(6,-1)$

$y = m(x-p)+q$

$y = \frac{-2}{3}(x-6) - 1$

$y = \frac{-2x}{3} + \frac{2\cdot 6}{3} - 1$ → 4

$y = \frac{-2x}{3} + 3$

2. Given the circle $x^2 + y^2 = 16$, state its center and radius: Center (0,0) radius is 4

$\sqrt{r^2} = \sqrt{16}$

$\boxed{r = 4}$

3. The point (6,-3) is on a circle that has its centre at (0, 0). Find the equation of the circle.

x y h k

Step1

$(x-h)^2 + (y-k)^2 = r^2$

$(6-0)^2 + (-3-0)^2 = r^2$

$36 + 9 = r^2$

$\sqrt{45} = \sqrt{r^2}$

$r = \sqrt{45}$

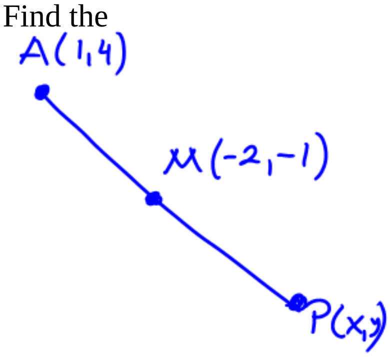
Step2

$x^2 + y^2 = (\sqrt{45})^2$

OR

$x^2 + y^2 = 45$

4. If the midpoint of a line segment is at M(-2, -1) and one endpoint is at A(1, 4). Find the coordinates of the other endpoint, P.



$2 \cdot \frac{(x+1)}{2} = -2 \cdot 2$

$x + 1 = -4$

$x = -4 - 1$

$\boxed{x = -5}$

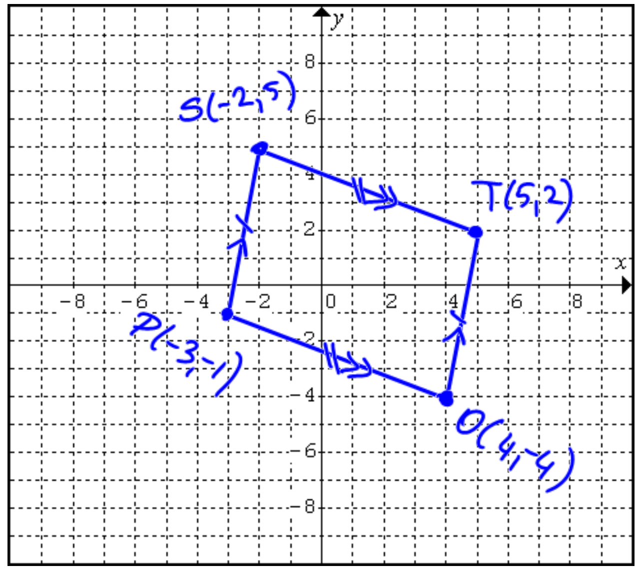
$2 \cdot \frac{(y+4)}{2} = -1 \cdot 2$

$y + 4 = -2$

$\boxed{y = -6}$

$\therefore P(-5, -6)$

5. A quadrilateral has vertices S(-2, 5), T(5,2), O(4,-4), & P(-3,-1). Use all applicable skills from this unit to determine what type of quadrilateral "STOP" is – be as specific as possible. Show ALL your calculations.
What's your plan? IE: What do you need to calculate?



Plan: calculate the length and slope of each line segment

Step1
$\overline{PS} = \sqrt{(-1-5)^2+[-3-(-2)]^2} = \sqrt{36+1} = \sqrt{37}$

$\overline{OT} = \sqrt{(4-5)^2+(-4-2)^2} = \sqrt{1+36} = \sqrt{37}$

$\overline{ST} = \sqrt{(-2-5)^2+(5-2)^2} = \sqrt{49+9} = \sqrt{58}$

$\overline{PO} = \sqrt{(-3-4)^2+[-1-(-4)]^2} = \sqrt{49+9} = \sqrt{58}$

$\overline{PS} = \overline{OT}$

$\overline{ST} = \overline{PO}$

Step2
$m_{PS} = \frac{-1-5}{-3-(-2)} = \frac{-6}{-1} = 6$

$m_{OT} = \frac{-4-2}{4-5} = \frac{-6}{-1} = 6$

$m_{ST} = \frac{2-5}{5-(-2)} = \frac{-3}{7}$

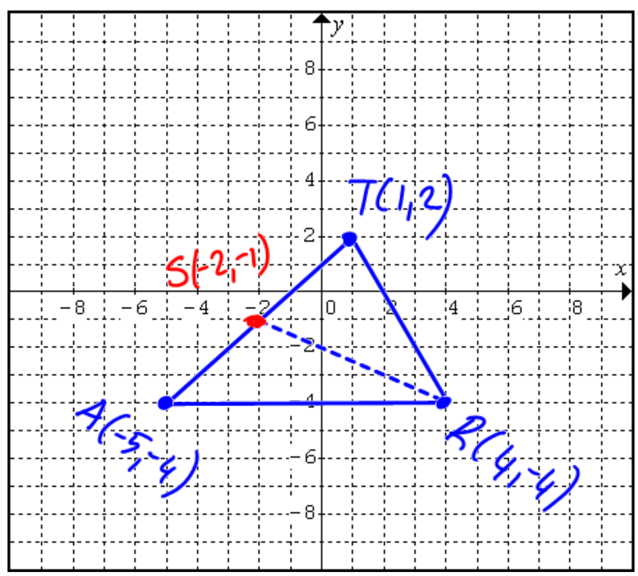
$m_{PO} = \frac{-4-(-1)}{4-(-3)} = \frac{-3}{7}$

$m_{PS} = m_{OT}$ $\overline{PS} \parallel \overline{OT}$

$m_{ST} = m_{PO}$

Conclusion
Since there are 2 pairs of parallel and equal sides, it's a parallelogram.

6. A triangle has vertices with coordinates R(4, -4), A(-5, -4) and T(1, 2). Find the equation of the median from R to the midpoint of side TA.
What's your plan? IE: What do you need to calculate?



Step1: Midpoint of $\overline{TA}$
$S(x,y) = \left(\frac{-5+1}{2}, \frac{-4+2}{2}\right) = (-2,-1)$

Step2: $m_{SR} = \frac{-4-(-1)}{4-(-2)} = \frac{-3}{6} = -\frac{1}{2} = -0.5$

Step3 $m = -0.5$ $S(-2,-1)$

$y = m(x-p)+q$

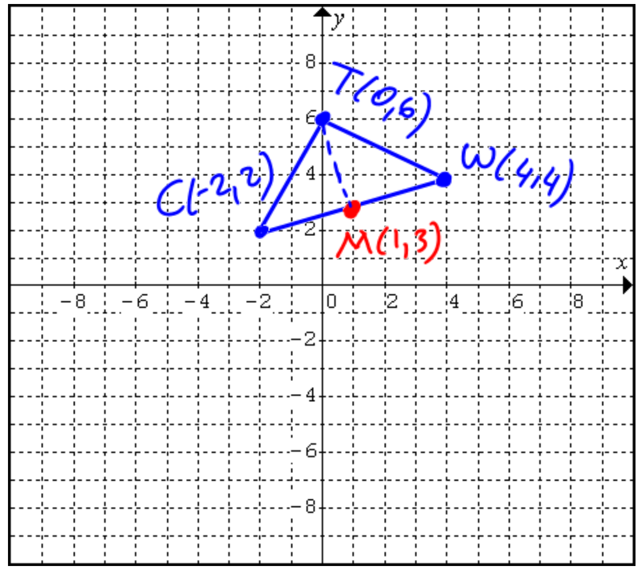
$y = -0.5[x-(-2)]+(-1)$

$y = -0.5(x+2)-1$

$y = -0.5x-1-1$

$\boxed{\therefore y = -0.5x - 2}$

7. A right triangle has vertices C(-2, 2), T(0, 6), W(4,4). Verify that the midpoint of the hypotenuse is equidistant from all three vertices.
What's your plan? IE: What do you need to calculate?



$\overline{CW}$ is the hypotenuse.

$M_{CW} = \left(\frac{-2+4}{2}, \frac{2+4}{2}\right) = (1,3)$

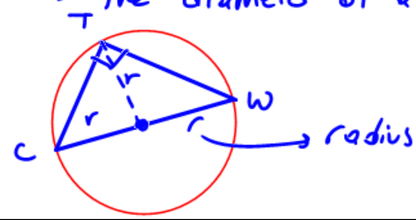
$\overline{CM} = \sqrt{(-2-1)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$\overline{MW} = \sqrt{(4-1)^2 + (4-3)^2} = \sqrt{9+1} = \sqrt{10}$

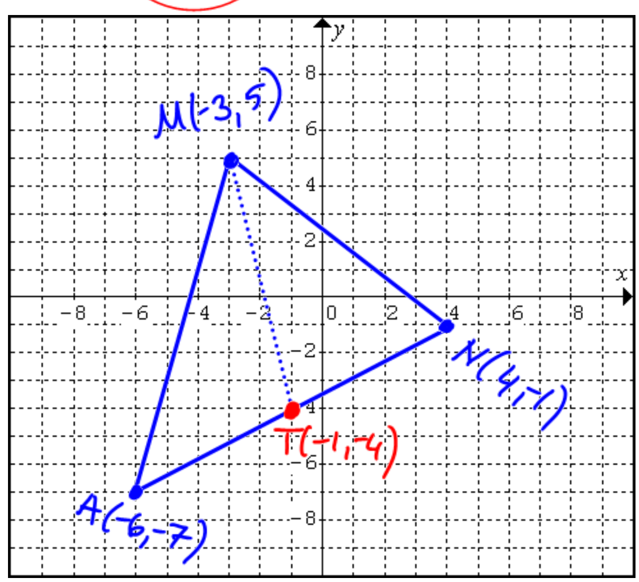
$\overline{TM} = \sqrt{(1-0)^2 + (3-6)^2} = \sqrt{1+9} = \sqrt{10}$

$\overline{CM} = \overline{MW} = \overline{TM}$

NOTE: $\overline{CW}$ is the diameter of a circle.



8. ΔMAN has with vertices at M(-3, 5), A(-6, -7), N(4, -1). Determine the length of the median from M to AN.
What's your plan? IE: What do you need to calculate?



Step1: Midpoint of $\overline{AN}$

$T(x,y) = \left(\frac{-6+4}{2}, \frac{-7+(-1)}{2}\right) = (-1,-4)$

Step2: $\overline{MT} = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2}$

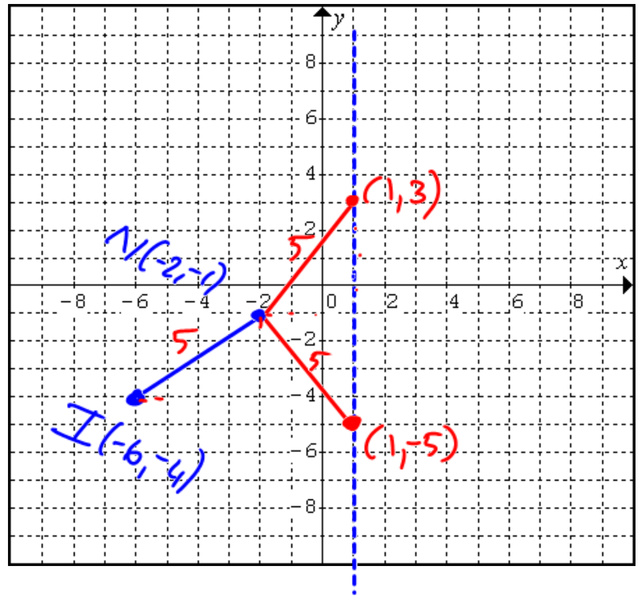
$= \sqrt{(-3+1)^2 + (5+4)^2}$

$= \sqrt{4+81}$

$= \sqrt{85}$

∴ The length of the median from M to AN is $\sqrt{85}$

9. Line segments IN and ON are equidistant. (same length) Determine the values of a if I(-6, -4), N(-2, -1), and O(1, a).



$\overline{IN} = \sqrt{[-6-(-2)]^2 + [-4-(-1)]^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\overline{ON} = \sqrt{(-2-1)^2 + (-1-a)^2}$

$(5)^2 = \left(\sqrt{9 + (-1-a)^2}\right)^2 \rightarrow$ square each side

$25 = 9 + (-1-a)^2$

$\sqrt{16} = \sqrt{(-1-a)^2} \longrightarrow$ square root each side

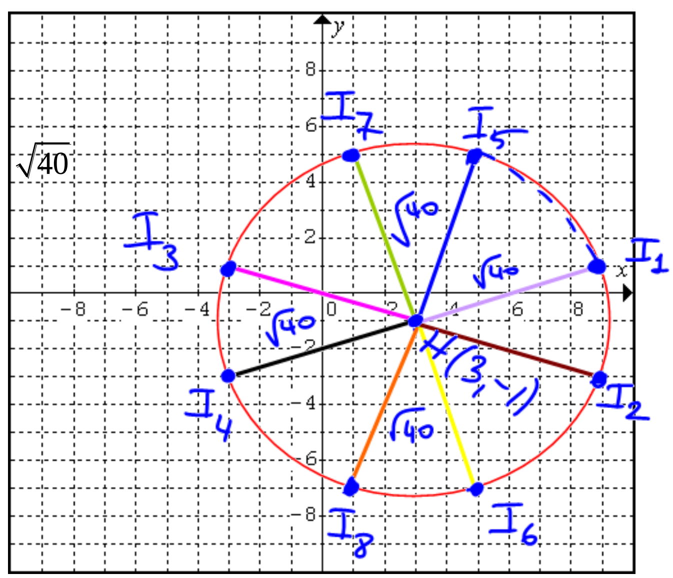
$4 = -1 - a$ OR $-4 = -1 - a$

$5 = -a$ $\quad -3 = -a$

$\boxed{a = -5}$ $\quad \boxed{a = 3}$

$\therefore a = -5, 3$

10. Line segment HI has H(3, -1). The length of HI is $\sqrt{40}$. What could the coordinates of I be?



$I(x, y)$

$HI = \sqrt{(x-3)^2 + (y+1)^2}$

$(\sqrt{40})^2 = \left(\sqrt{(x-3)^2 + (y+1)^2}\right)^2$ square each side

$40 = (x-3)^2 + (y+1)^2$

$36 + 4 \rightarrow ①$

$4 + 36 \rightarrow ②$

Scenario 1

$\sqrt{(x-3)^2} = \sqrt{36}$ $\quad \sqrt{(y+1)^2} = \sqrt{4}$

$x - 3 = 6$ $\quad y + 1 = 2$

$\boxed{x = 9}$ $\quad \boxed{y = 1}$

$x - 3 = -6$ $\quad y + 1 = -2$

$\boxed{x = -3}$ $\quad \boxed{y = -3}$

$I_1(9,1)$
$I_2(9,-3)$
$I_3(-3,1)$
$I_4(-3,-3)$

Scenario 2

$\sqrt{(x-3)^2} = \sqrt{4}$ $\quad \sqrt{(y+1)^2} = \sqrt{36}$

$x - 3 = 2$ $\quad y + 1 = 6$

$\boxed{x = 5}$ $\quad \boxed{y = 5}$

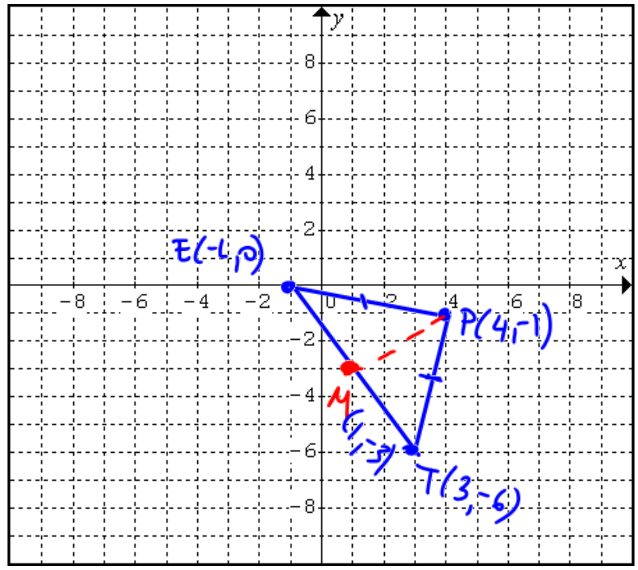
or $\quad$ or

$x - 3 = -2$ $\quad y + 1 = -6$

$\boxed{x = 1}$ $\quad \boxed{y = -7}$

$I_5(5,5)$
$I_6(5,-7)$
$I_7(1,5)$
$I_8(1,-7)$

11. ΔPET is isosceles with vertices at P(4, -1), E(-1, 0), T(3, -6). The two equal sides are PE and PT. Verify that the median from P to ET is also an altitude.
What's your plan? IE: What do you need to calculate?



Step1: $M_{ET}(x,y) = \left(\frac{-1+3}{2}, \frac{0+(-6)}{2}\right) = (1,-3)$

Step2: Slope of ET

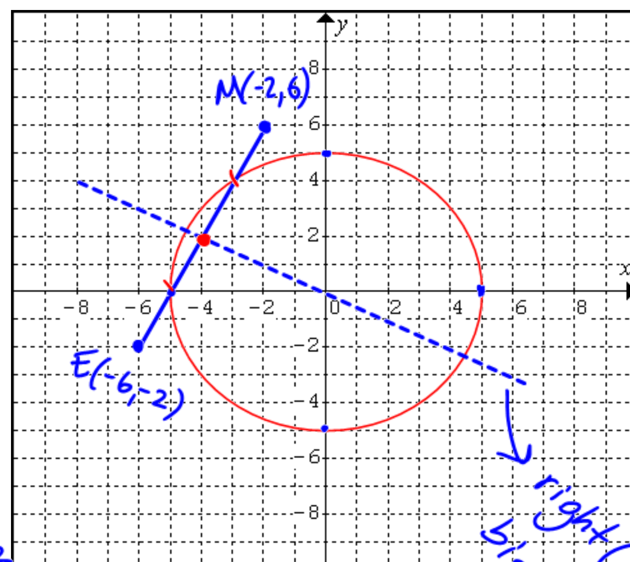
$m_{ET} = \frac{-6-0}{3-(-1)} = \frac{-6}{4} = -3/2$

$m_{MP} = \frac{-1-(-3)}{4-1} = \frac{-1+3}{3} = 2/3$

m_{ET} is opp. reciprocal of m_{MP}; therefore they're perpendicular. Line segment MP is an altitude.

r^2

12. A circle with the equation $x^2 + y^2 = 25$ has its centre at (0, 0). A chord of the circle has its end points at M(-2, 6) and E(-6, -2). Determine the equation of the perpendicular bisector of ME and verify that the perpendicular bisector passes through the centre of the circle.
What's your plan? IE: What do you need to calculate?



right (perpendicular) bisector

1) $x^2+y^2=5^2 \to r^2=5^2 \to \boxed{r=5}$ Point

midpoint ← 2) $T(x,y) = \left(\frac{-6+(-2)}{2}, \frac{6+(-2)}{2}\right) = (-4,2)$

3) $m_{EM} = \frac{-2-6}{-6-(-2)} = \frac{-8}{-4} = 2$ $m_{bisector} = -1/2$

4) $m_{bisector} = -1/2$ T(-4,2)

$y = \frac{-1}{2}[x-(-4)]+2$

$y=-0.5(x+4)+2$

$y=-0.5x-2+2$

$\boxed{y=-0.5x \text{ OR } y=-\frac{1}{2}x}$

VERIFICATION

Center (0, 0)

LS	RS
y	-0.5x
0	0

LS = RS ∴ Center is on the right bisector

Answers:

1a. $\sqrt{117}$ =10.8 b. (1.5, 2) c. $\frac{-2}{3}$ d. $y=\frac{-2}{3}x+3$ or $2x+3y-9=0$ 2. (0, 0); 4 3. $x^2+y^2=45$ 4. (-5, -6)

5. opposite sides: 7.6 and 6.1; opposite slopes: $\frac{-3}{7}$ and 6; parallelogram 6. $y=\frac{-1}{2}x-2$ or $x+2y+4=0$ 7. all distances are $\sqrt{10}$

8. $\sqrt{85}$ = 9.2 9. a is -5 or 3 10. (1, 5); (5, 5); (9, 1); (9, -3); (5, -7); (1, -7); (-3, -3); or (-3, 1)

11. the slope of the median is $\frac{2}{3}$ and the slope of ET is $\frac{-3}{2}$ so they're opposite reciprocals 12. $y=\frac{-1}{2}x$