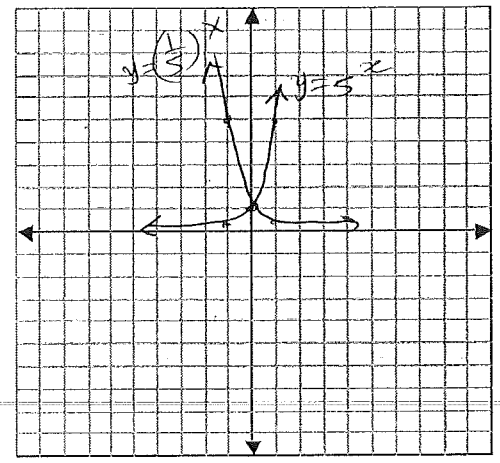
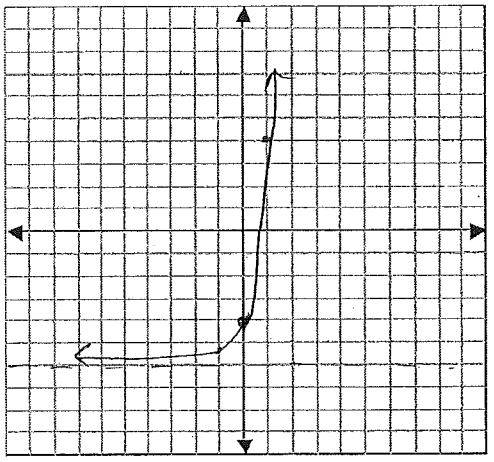
Unit 4 Review:

1. Graph $y = 5^x$ and $y = (1/5)^x$ on the same grid.



2. For the curve $y = 2(5^x) - 6$, determine
   a) the transformations of $y = 5^x$
      - vertically stretched by a factor of 2
      - vertical translation 6 units down.

   b) the horizontal asymptote $y=-6$
   c) the y-intercept -4 [sub $x=0$].
   d) whether the curve is increasing or decreasing increasing.
   e) the domain: $\{x \in \mathbb{R}\}$
   range: $\{y \in \mathbb{R} \mid y > -6\}$
   f) sketch the graph



3. Repeat the above for the curve $y = 5(\frac{1}{2})^x + 3$.

HA: $y=3$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y > 3\}$

decreasing.

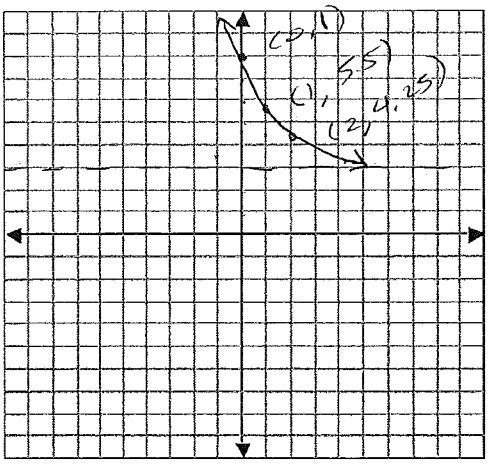
y-int: 8

vertically stretched by a factor

$(-1, 2) \rightarrow (-1, 13)$

$(0, 1) \rightarrow (0, 8)$

$(1, \frac{1}{2}) \rightarrow (1, 5.5)$



4. List 2 equivalent powers for each exponential function below.

a) $y = 25^x$

$25 = 5^2$ or $25 = \left(\frac{1}{25}\right)^{-1}$

$\therefore y = 5^{2x}$ $\therefore y = \left(\frac{1}{25}\right)^{-x}$

b) $y = \left(\frac{1}{8}\right)^x$ $\frac{1}{8} = \frac{1}{2^3}$

$\therefore y = \left(\frac{1}{2}\right)^{3x}$ or $\frac{1}{8} = 2^{-3}$

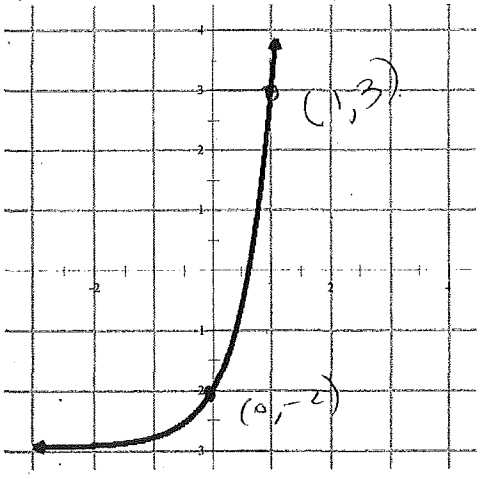
$\therefore y = 2^{-3x}$

5. The exponential function $y = \left(\frac{1}{7}\right)^x$ is vertically stretched by a factor of 2, horizontally compressed to $\frac{1}{4}$, translated right 3 units and translated up 5 units. State the transformed equation for this exponential function.

$y = 2\left(\frac{1}{7}\right)^{4(x-3)} + 5$

6. Following are the graphs of some exponential functions with equations of the form $y = b^x$, $b>0$. Using what you have learned about exponential functions and without the aid of your calculator, write the equation of each function.

a)



$y = 6^x - 3$.

HA: $y = -3$ $\therefore c = -3$.

$y = ab^x - 3$.

Sub $x=0$, $y=-2$:

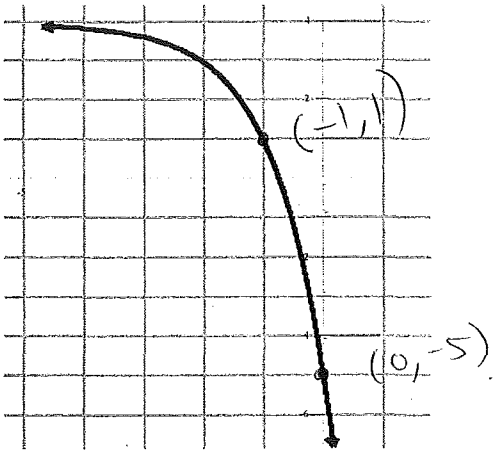
$-2 = a - 3$

$a = 1$ $\therefore$ $y = b^x - 3$

Sub $(1,3)$ $b = 6$

$\therefore y = 6^x - 3$

b)



$y = -\left(3\right)^{x+2} + 4$ or $y = -9(3)^x + 4$

HA: $y = 4$ $\therefore c = 4$ reflected.

$\therefore y = -a(b)^x + 4$

Sub $(0,-5) \Rightarrow -5 = -a + 4$

$a = 9$

sub $(-1,1) \Rightarrow y = -9(b)^x + 4$

$1 = -9(b^{-1}) + 4$

$b = 3$ $\therefore y = -9(3)^x + 4$

7. A rare coin was bought for $1200. Its value increases by 5% each year. Determine the coin's value ten years after it was bought.

$y = 1200(1-0.05)^n$

$= 1200(0.95)^n$

After 10 years: $y = 1200(0.95)^{10} = 1954.67 \approx^{\#} 1954.65$

8. A small country in Africa must maintain at least 5020 acres of forest at all times. If the government wishes to cut down 11% of the forest every year for the next 4 years, how many acres of forest must the country start with to ensure they abide by the law?

$y = a(1-0.11)^n$

$5020 = a(0.89)^4$

$a = \frac{5020}{0.894} = 8001$ acres

9. Thorium-227 has a half-life of 18.4 days. How much time will a 50-mg sample take to decompose to 10 mg? (use "guess and check"). $A = 50\left(\frac{1}{2}\right)^{\frac{t}{18.4}}$

$10 = 50\left(\frac{1}{2}\right)^{\frac{t}{18.4}}$ guess and check $t = 42.7$ days.

10. Solve the equation $2^{2x} - 5(2^x) + 4 = 0$

$p^2 - 5p + 4 = 0 \quad [p = 2^x]$

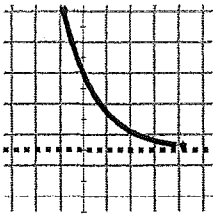
$(p-4)(p-1) = 0 \quad \therefore 2^x = 4 \text{ OR } 2^x = 1$

$p = 4 \quad p = 1 \qquad x = 2 \qquad x = 0.$

Solutions:

2. VS by 2, down 6; y = -6, (0, -4), increasing, $D: \{x \in \mathbb{R}\}$, $R: \{y \in \mathbb{R} \mid y > -6\}$

3. VS by 5, up 3; y = 3, (0, 8), decreasing, $D: \{x \in \square\}$, $R: \{y \in \square \mid y > 3\}$



4.a) $y = 5^{2x}$, $y = (1/25)^{-x}$, etc. b) $y = (\frac{1}{2})^{3x}$, 8^{-x}, etc.

5. $2(1/7)^{4(x-3)} + 5$

6 .a) $6^x - 3$ b) $-3^{x+2} + 4$

7. $1954.67

8. 8001 acres

9. 42.7 days

Exponential Equations and Rational Exponents Review.

1. Solve the following exponential equations. Show all steps.

a) $4^{2x+5} = \frac{1}{64}$

$4^{2x+5} = 4^{-3}$

$2x+5 = -3$

$2x = -8$

$x = -4$

b) $27^{x-6} = 9^{2x+1}$

$(3^3)^{(x-6)} = (3^2)^{(2x+1)}$

$3x-18 = 4x+2$

$3x-4x = 2+18$

$-x = 20$

$\boxed{x = -20}$

c) $3(5)^{x^2-x} = 75$

$5^{x^2-x} = 25 = 5^2$

$x^2-x-2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad or \quad x = -1$

d) $32^{(2x+1)}(8^{4x}) = \left(\frac{1}{64}\right)^{2-x}$

$(2^5)^{2x+1}(2^3)^{4x} = (2^{-6})^{2-x}$

$10x+5+12x = -12+6x$

$16x = -17$

$\boxed{x = \frac{-17}{16}}$

2. A standard can of Red Bull contains about $80\ mg$ of caffeine. Every 5 hours, the mass of caffeine in an adult's bloodstream reduces by half. While studying for a test, Roger drank a can of Red Bull at 11:00 PM last night. Determine the mass of caffeine in Adam's bloodstream at 2 PM today. $A = 80\left(\frac{1}{2}\right)^{\frac{t}{5}}$ $t = 15$

$= 80\left(\frac{1}{2}\right)^{\frac{15}{5}} = 80\left(\frac{1}{2}\right)^3$

$= 80\left(\frac{1}{8}\right) = 10\ mg.$

3. Simplify.

a) $(-4x^{-7}m^3)^{-3}\left(\frac{2x^6m^{-4}}{x^9m^{-1}}\right)^5$

$= \frac{(-4)^{-3}x^{21}m^{-9}\,2^5x^{30}m^{-20}}{x^{45}m^{-5}}$

$= \frac{32x^{51}m^{-29}}{-64x^{45}m^{-5}} = \frac{-1x^6}{2m^{24}}$

b) $\frac{\left(81x^{\frac{3}{8}}m^{\frac{1}{3}}\right)^{\frac{1}{4}}}{(-27x^{-6}m^{10})^{\frac{1}{3}}} = \frac{3x^{\frac{3}{32}}m^{\frac{1}{12}}}{-3x^{-2}m^{\frac{10}{3}}} = -x^{\frac{3}{32}+2}m^{\frac{1}{12}-\frac{10}{3}}$

$= -\frac{x^{\frac{67}{32}}}{m^{\frac{39}{12}}}$

4. Evaluate. $4^{-1}(3^{-2}+2^4)^{-2}$

$= \frac{1}{4}\left(\frac{1}{9}+16\right)^{-2} = \frac{1}{4}\left(\frac{1+144}{9}\right)^{-2} = \frac{1}{4}\left(\frac{9}{145}\right)^2 = \frac{81}{84100}$

Answers

1. a. $x = -4$ b. $x = -20$ c. $x = -1, x = 2$ d. $x = -\frac{17}{16}$

2. 10 mg. 3. a. $-\frac{x^6}{2m^{24}}$ b. $-\frac{x^{\frac{67}{32}}}{m^{\frac{39}{12}}}$ c. $\frac{81}{84100}$

Finding Equation of an Exponential Function:

1. Find the exponential function through (2, 10) and (4, 22) that has a horizontal asymptote at y = 4. (Equation of the form: $y = ab^x + c$)

$y = ab^x + 4$

sub in (2, 10) and (4, 22): $10 = ab^2 + 4 \Rightarrow 6 = ab^2$ $\quad 22 = ab^4 + 4 \Rightarrow 18 = ab^4$

$\therefore \frac{ab^4}{ab^2} = \frac{18}{6}$

$b^2 = 3$

$b = \sqrt{3}$

$6 = a(\sqrt{3})^2 \Rightarrow a = \frac{6}{3} = 2$

$\therefore y = 2(\sqrt{3})^x + 4$

2. Find an exponential function that passes through (3, 12.5) and (4, 11.25) and has a horizontal asymptote of y = 10. (Equation of the form: $y = ab^x + c$)

$y = ab^x + 10$

$12.5 = ab^3 + 10$ $\quad 11.25 = ab^4 + 10$

$2.5 = ab^3$ $\quad 1.25 = ab^4$

$\frac{ab^4}{ab^3} = \frac{1.25}{2.5}$

$b = \frac{1}{2}$

$\therefore 2.5 = a(\frac{1}{2})^3 \Rightarrow 2.5 = \frac{a}{8}$

$a = 20$

$y = 20(\frac{1}{2})^x + 10$

3. The graph of $f(x) = 2^x$ is compressed vertically by a factor of $\frac{1}{2}$ reflected in the y-axis, and translated right 4 units and downward 5 units.

a) Write the equation of the new function. $y = \frac{1}{2}(2)^{-(x-4)} - 5$

b) State the domain, range, y-intercept and equation of the horizontal asymptote.

$D = \{x \in \mathbb{R}\}$ $\quad$ y-int: 3

$R = \{y \in \mathbb{R} \mid y > -5\}$ $\quad$ HA: $y = -5$

4. The equation of the function that represents $f(x) = (\frac{1}{4})^x$ after it is compressed horizontally by a factor of $\frac{1}{2}$, reflected in the x-axis, and shifted 4 to the left and 6 units up.

a) Write the equation of the new function. $y = -\left(\frac{1}{4}\right)^{2(x+4)} + 6$

b) State the domain, range, and equation of the horizontal asymptote.

D: $\{x \in \mathbb{R}\}$ $\quad$ HA: $y = 6$.

R: $\{y \in \mathbb{R} \mid y < 6\}$

Textbook Review: p. 267 #3ad, 4c, 5d, 7f, 8acd, 11d, 12, 14d, 15c, 16b, 17abcd