Part 1 – Exponential Growth

The exponential function can be used as a model to solve problems involving exponential growth.

$$A(x) = a(1+b)^x$$

where: $f(x) =$ final amount or number
$a =$ initial amount
$b =$ growth rate in decimal
$x =$ the number of growth period

Ex1. The population of a small town increases by 3% every year. Its population in 1996 was 1250.

b a

a) Find an expression to represent the population of the town n years after 1996.

$b = 3\% = 0.03$
$a = 1250$

$P = 1250(1+0.03)^n$
$P = 1250(1.03)^n$

b) Determine the population in the year 2017.

$n = 2017 - 1996$
$n = 21$

$P = 1250(1.03)^{21}$
$\doteq 2325$

∴ The population is 2325.

c) In what year will the population reach 3000 people?

We're given the population. Sub 3000 for P to solve n.

$P = 1250(1.03)^n$

$\frac{3000}{1250} = \frac{1250(1.03)^n}{1250}$ divide each side by 1250 to simplify

$2.4 = (1.03)^n$ to figure out "n" we need to use trial & error method.

Let's try 20 for n

$(1.03)^{20} = 1.8$
$(1.03)^{25} = 2.09$
$(1.03)^{30} = 2.4$

∴ It'll reach 3000 after 30 years. (Year 2026)

EXPONENTIAL DOUBLING

Bacterial and viral cultures are examples of substances that grow at a rate which is exponential in nature – they double over a given period of time.

In general, these cultures grow according to the following exponential equation:

$$\boxed{A(x) = a(2)^{t/d}}$$

where $A(x)$ is the total amount or number
a is the initial amount or number
t is the elapsed time
d is the doubling period

Ex1. One bacterium divides into two bacteria every 5 days. Initially, there are 15 bacteria. How many bacteria will there be in 30 days?

$a = 15$
$t = 30$ days
$d = 5$ days

$$P = a(2)^{t/d}$$
$$= 15(2)^{30/5}$$
$$= 15(2)^{6}$$
$$= 15(64)$$
$$= 960$$

$\therefore$ There'll be 960 bacteria.

Ex2. A bacterial culture starts with 3000 bacteria and grows to a population of 12 000 after 3 hours.

a) Find the doubling period.

$a = 3000$
$P = 12000$
$t = 3$ hours
$d = ?$

$$P = a(2)^{t/d}$$
$$\frac{12000}{3000} = \frac{3000(2)^{3/d}}{3000}$$
$$4 = 2^{3/d}$$

If $2^2 = 2^{3/d}$

then $d \cdot 2 = \frac{3}{d} \cdot d$

$2d = 3$

$d = 3/2$

$\therefore$ The doubling period is 1.5 hours.

b) Find an expression to represent the population after *t* hours.

$$P = 3000(2)^{t/1.5}$$

c) Determine the number of bacteria after 8 hours.

$$P = 3000(2)^{8/1.5}$$

$= 120952$

be careful. Put the numbers into your calc:

$3000 \boxed{\times} 2 \boxed{\wedge} (8 \div 1.5)$

$\therefore$ There'll be 120952 bacteria in 8 hours.

1. The population of a city is 810 000. If it is increasing by 4% per year, estimate the population in four years.

$f(x) = a(1+b)^x$

$a = 810{,}000$
$b = 4\% = 0.04$
$x = 4$
$P(x) = ?$

$P(4) = 810{,}000(1+0.04)^4$
$= 810000(1.04)^4$
$= 947585$

$\therefore$ The population will be 947585 in 4 years.

2. A painting, purchased for $10 000 in 1990, increased in value by 8% per year. Find the value of the painting in the year 2000.

$f(x) = a(1+b)^x$

$a = \$10000$
$b = 8\% = 0.08$
$x = 2000 - 1990 = 10$
$V = ?$

$V(10) = 10000(1+0.08)^{10}$
$= 10000(1.08)^{10}$
$= \$21589.25$

$\therefore$ The value of the painting will be $21589.25 in 10 years.

3. A river is stocked with 5000 salmon. The population of salmon increases by 7% per year.
a. Write an expression for the population t years after the salmon were put into the river.
b. What will the population be in 3 years? 15 years?
c. How many years does it take for the salmon population to double?

a) $f(x) = a(1+b)^x$

$a = 5000$
$b = 7\% = 0.07$
$x = t$

$P(x) = 5000(1+0.07)^t$
$P(x) = 5000(1.07)^t$

b) $P(3) = 5000(1.07)^3$
$= 6125$

$\therefore$ The pop. will be 6125 in 3 years

$P(15) = 5000(1.07)^{15}$
$= 13795$

$\therefore$ The pop will be 13795 in 15 years.

c) $P(x) = 5000(1.07)^t$

$$\frac{10000}{5000} = \frac{5000(1.07)^t}{5000}$$

$2 = (1.07)^t$

when $t = 10$ it's 1.97
$t = 11$ it's 2.10
$t = 10.5$ it's 2.03
$t = 10.2$ it's 1.99

$\therefore$ It'll take about 10.2 years.

4. A house was bought 6 years ago for \$175 000. If real-estate values have been increasing at the rate of 4% per year, what is the value of the house now?

$f(x) = a(1+b)^x$ $a = \$175000$
$b = 4\% = 0.04$
$x = 6$
$V(6) = ?$

$V(6) = 175000(1+0.04)^6$
$= 175000(1.04)^6$
$= \$221430.83$

$\therefore$ The value of the house is \$221,430.83 now.

5. If a bacteria population doubles in 5 d,
a. When will it be 16 times as large?
b. When was it $\frac{1}{2}$ of its present population?
c. When was it $\frac{1}{4}$ of its present population?
d. When was it 1/32 of its present population?

a) $P = a(2)^{t/d}$

$$\frac{16a}{a} = \frac{a(2)^{t/5d}}{a}$$

$16 = 2^{t/5d}$

if $2^4 = 2^{t/5d}$

then $4 = \frac{t}{5d}$ cross multiply

$\boxed{t = 20d}$

b) $P = a(2)^{t/5d}$

$\frac{1}{2}a = a(2)^{t/5d}$

$\frac{1}{2} = (2)^{t/5d}$

if $2^{-1} = 2^{t/5d}$

then $-1 = \frac{t}{5d}$ cross mult.

$-5d = t$ $\therefore$ 5 days ago
$t = -5d$

c) $\frac{1}{4}a = a(2)^{t/5d}$

$\frac{1}{4} = (2)^{t/5d}$

$2^{-1} = (2)^{t/5d}$

if $2^{-2} = 2^{t/5d}$

then $-2 = \frac{t}{5d}$ cross x

$\boxed{-10d = t}$ $\therefore$ 10 days ago.

6. The population of a city was estimated to be 125 000 in 1930 and 500 000 in 1998.

a. Estimate the population of the city in 2020.

b. If the population continues to grow at the same rate, when will the population reach 1 million? (in what year)

a). $f(x) = a(1+b)^x$

Let "1930" be the starting year; therefore 125000 will be the initial population.

$a = 125000$

$f(x) = 500000$

$x = 1998 - 1930 = 68$ years.

find the growth % (b)

$$\frac{\cancel{500000}\,4}{\cancel{125000}} = \frac{\cancel{125000}(1+b)^{68}}{\cancel{125000}}$$

$$4 = (1+b)^{68}$$

$$\sqrt[68]{4} = \sqrt[\cancel{68}]{(1+b)^{\cancel{68}}}$$

$$\sqrt[68]{2^2} = (1+b)$$

$$2^{2/68} = (1+b)$$

$$2^{1/34} = (1+b)$$

$$P(x) = a\left(2^{1/34}\right)^{x}$$

$2020 - 1930 = 90$

$$= 125000(2)^{\frac{90}{34}}$$

$$= \underline{\underline{782986}}$$