MCR3U **UNIT T 7 REVIEW– SEQUENCES AND SERIES**

$S_n = \frac{rt_n - t_1}{r-1} = a[\frac{r^n - 1}{r-1}]$	$S_n = \frac{n}{2}[t_1 + t_n] = \frac{n}{2}[2a + (n-1)d]$

1. Beside each sequence, write the next term and then state whether it is arithmetic, geometric, or neither.

a) 100, 50, 25, . . . 12.5 geometric b) 50, 60, 70, . . . 80 arithmetic

c) 1, 2, 3, 5, . . . 8 neither d) 1, 8, 15, . . . 22 arithmetic

2. Write a general formula for the sequence -3, -8, -13, -18, . . .

$t_n = a + (n-1)d$

$= -3 + (n-1)(-5) = -5n + 2$

3. Write a recursive formula for the sequence 2, 3, 6, 18, 108, ...

$t_n = (t_{n-1})(t_{n-2})$, $t_1 = 2$

$t_2 = 3$

4. Martina has won the grand prize in "Cash Forever" lottery. She will receive $5,000 the first year and receive an annual increase of $300 each year, on top of the $5,000 annual payment.

a. Write the first 4 terms of the sequence of her annual payment.

5000, 5300, 5600, 5900,

b. Determine the general term of this sequence. Find her annual payment in the tenth year.

$t_n = 5000 + (n-1)(300)$

$= 300n + 4700$

$t_{10} = (300)(10) + 4700$

$= 7700$ ∴ Her 10^{th} year payment was $7700.

c. In which year should Martina expect her payment to be $10,400? (Use appropriate concept(s) learned in this chapter to answer this question)

$t_n = 300n + 4700$

$10400 = 300n + 4700$

$300n = 5700$

$n = \frac{5700}{300}$

$= 19$

∴ In the 19^{th} year, payment will be $10 400

d. Is there a year where Martina's payment will be $20,000? Explain.
(Use appropriate concept(s) learned in this chapter to answer this question)

$20000 = 300n + 4700$

$n = 51$

∴ Since 'n' is a positive whole number $(n \in \mathbb{N})$
Martina's 51^{st} payment will be $20000.

5. A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. Write the general term and use it to find how high (*to the nearest tenth of a foot*) does the ball bounce on the fifth bounce.

after first bounce: $8(0.8) = 6.4$

$\therefore t_1 = 6.4 \quad r = 0.8$

$t_n = ar^{n-1}$

$= (6.4)(0.8)^{n-1}$

$\therefore t_5 = 6.4(0.8)^4$

$= 2.6 \text{ ft}$

$\therefore$ The height after 5th bounce will be 2.6 ft.

6. Explain how you can determine number of terms for the sequence: 6, 17, 28, 39, … , 435? Find n.

Use the general term for appropriate sequence (arith / geo)

In this case: arithmetic $\Rightarrow$ $6 + (n-1)(11) = t_n$ $\rightarrow$ is the last term

$6 + (n-1)(11) = 435$

$n = 40$

7. If $t_4 = 54$ *and* $t_7 = 1458$ in a geometric sequence, determine the value of 'a' and 'r'.

$t_n = ar^{n-1}$

$\boxed{54 = ar^3}$ (2) $\boxed{1458 = ar^6}$ (1)

$\frac{(1)}{(2)}: \quad \frac{1458}{54} = \frac{ar^6}{ar^3}$

$27 = r^3$ ← cube root

$\boxed{r = 3}$

Sub in $r = 3$ in (2)

$54 = a(3)^3$

$54 = 27a$

$\boxed{a = 2}$

8. Find S_{11} for the following series.

a) $-90 - 82 - 74 + \ldots$ $a = -94$ $d = 8$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{11} = \frac{11}{2}[-180 + 10(8)]$

$= -550$

b) $840 - 420 + 210 - \cdots$ $a = 840$ $r = -0.5$

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

$= 840\left(\frac{(-0.5)^{11} - 1}{-0.5 - 1}\right) = 560\frac{35}{128}$

9. Evaluate

a) $3 + 6 + 12 + \cdots + 768$ ← geo.

$S_n = \frac{r(\text{last term}) - \text{first term}}{r - 1}$

$= \frac{(2)(768) - 3}{1}$

$= 1533$

or find n' then use $S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

b) $11 + 14 + 17 + \cdots + 134$

$134 = 11 + (n-1)(3)$

$n = 42$

$S_{42} = \frac{42}{2}(t_1 + t_n)$

$= 21(11 + 134)$

$= 3045$

10. A lottery plans to give out \$5,000,000 in prizes. The first ticket drawn wins \$20, the second ticket drawn wins \$50, the third ticket drawn wins \$125, and so on. Can the lottery afford to give out 14 prizes?

20, 50, 125, ... Idea: Is $S_{14} < 5000000$

$$S_{14} = a\left(\frac{r^n - 1}{r-1}\right)$$

$$= 20\left(\frac{2.5^{14} - 1}{2.5 - 1}\right)$$

$$\doteq 4967,040.40$$

$\therefore S_{14} <$ 5 million

$\therefore$ Lottery can afford to give out 14 prizes.

11. What is difference between two formulas for geometric series?

$S_n = a\left(\frac{r^n-1}{r-1}\right) \rightarrow$ needs number of terms but not t_n.

$S_n = \frac{rt_n - t_1}{r-1}$ does not need n. Need last term.

12. The arithmetic series 13 + 17 + 21 ... + t_n has a sum of 931. How many terms does the series have?

$a = 13$ $d = 4$ $S_n = 931$

$$S_n = \frac{n}{2}(2a + (n-1)(d))$$

$$931 = \frac{n}{2}(26 + (n-1)(4))$$

$$931 = \frac{n}{2}(4n + 22)$$

$$931 = n(2n + 11)$$

$$2n^2 + 11n - 931 = 0$$

$$n = \frac{-11 \pm \sqrt{11^2 - 4(2)(-931)}}{2(2)}$$

$$= \frac{-11 \pm 87}{4}$$

$\cancel{\frac{-11-87}{4}}$

$\frac{-11+87}{4} = 19$.

$\therefore$ series has 19 terms

13. Expand each of the following:

a. $(3x-2)^5$

$= (3x)^5 + 5(3x)^4(-2) + 10(3x)^3(-2)^2 + 10(3x)^2(-2)^3 + 5(3x)(-2)^4 + (-2)^5$

$= 243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32$

b. $(x+2y)^6$

$= x^6 + 6x^5(2y) + 15x^4(2y)^2 + 20x^3(2y)^3 + 15x^2(2y)^4 + 6(x)(2y)^5 + (2y)^6$

$= x^6 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6$.

$t_n = a+(n-1)d$	$t_n = ar^{n-1}$	$S_n = \frac{n}{2}[2a+(n-1)d]$
$S_n = \frac{n}{2}[a+t_n]$	$S_n = \frac{a(r^n-1)}{r-1}$	

43. Find the formula for the nth term and find the indicated term for each arithmetic sequence.
a) 3, 5, 7, ... ;t_{30} b) −4, 3, 10, ... ;t_{18}

44. Find the number of terms in each arithmetic sequence.
a) 4, 9, 14, ... , 169 b) 19, 11, 3, ... ,−229

45. The Women's World Cup of Soccer tournament was first held in 1991. The next two tournaments were held in 1995 and 1999.
a) Write a formula for finding the year in which the nth tournament will be held.
b) Predict he year of the 35th tournament.

46. Find the formula for the nth term and find the indicated term for each geometric sequence.
a) 27, 9, 3, ... ;t_6 b) 1, −3, 9, ... ;t_7

47. Find $a,\ r,\ and\ t_n$ for each geometric sequence.
a) $t_4 = 24\ and\ t_6 = 96$ b) $t_2 = -6\ and\ t_5 = -162$

48. Use the recursion formula to write the first 5 terms of each sequence.
a) $t_1 = 3;\ t_2 = 3;\ t_n = t_{n-1} + t_{n-2}$ b) $f(1) = 8;\ f(n) = 0.5f(n-1)$

49. Identify whether the series is Arithmetic or Geometric. Then, find n.
a) $1+2+4+...+1024$ b) $-5-2+1+4...+133$ c) $16384+4096+...+1$

50. Find the indicated sum for each arithmetic series.
a) S_{25} for $-20-18-16$ b) $1+\frac{5}{4}+\frac{3}{2}+...+20$

51. The side lengths in a quadrilateral from an arithmetic sequence. The perimeter is 38 cm and the shortest side measures 5 cm. What are the other side lengths?

52. Find the indicated sum for each geometric series.
a) S_{12} for $4-8+16-32+...$ b) $3645-1215+405-...+5$

53. A ball is kicked from the ground 6.4 m into the air. The ball falls, rebounds to 60% of its previous height and falls again. If the ball continues to rebound and fall in this manner, find the total distance the ball travels until it hits the ground for the fifth time (assume the ball bounces vertically with no curvature in its path).