Unit 3 Review:

Asymptotes of Rational Functions

1. Find the equation of the horizontal asymptote of each curve.

(a) $f(x) = \dfrac{2x+3}{x+5}$

VA: $x = -5$

HA: $y = 2$

(b) $g(x) = \dfrac{x}{x^2-9}$

VA: $x = \pm 3$

HA: $y = 0$

(c) $h(x) = \dfrac{x^2-1}{x^2+1}$

VA: NONE

HA: $y = 1$

(d) $k(x) = 1 - \dfrac{x}{x^2-4}$

VA: $x = \pm 2$

HA: $y = 1 - 0 = \underline{1}$

2. Find an equation of the oblique asymptote of each curve.

(a) $y = \dfrac{2x^2-4x+6}{x}$

$y = (2x-4) + \dfrac{6}{x}$

oblique: $y = 2x-4$

(b) $h(x) = \dfrac{x^3-8}{x^2}$

$$\begin{array}{r}
x \\
x^2 \overline{)\, x^3+0x^2+0x-8} \\
\underline{x^3+0x^2+0x} \\
-8
\end{array}$$

$= x - \dfrac{8}{x^2}$

oblique: $y = x$

(c) $y = \dfrac{9x^2}{3x-1}$

$$\begin{array}{r}
3x+1 \\
3x-1 \overline{)\, 9x^2+0x+0} \\
\underline{9x^2-3x} \\
3x+0 \\
\underline{3x-1} \\
1
\end{array}$$

$\therefore$ $y = 3x+1$.

(d) $f(x) = \dfrac{x^3+6x^2+9x+10}{x^2+4}$

$$\begin{array}{r}
x+6 \\
x^2+4 \overline{)\, x^3+6x^2+9x+10} \\
\underline{x^3+6x^2+4x} \\
6x^2+5x+10 \\
\underline{6x^2+0x+24} \\
5x-14
\end{array}$$

$\therefore$ $y = x+6$ is the oblique asymptote.

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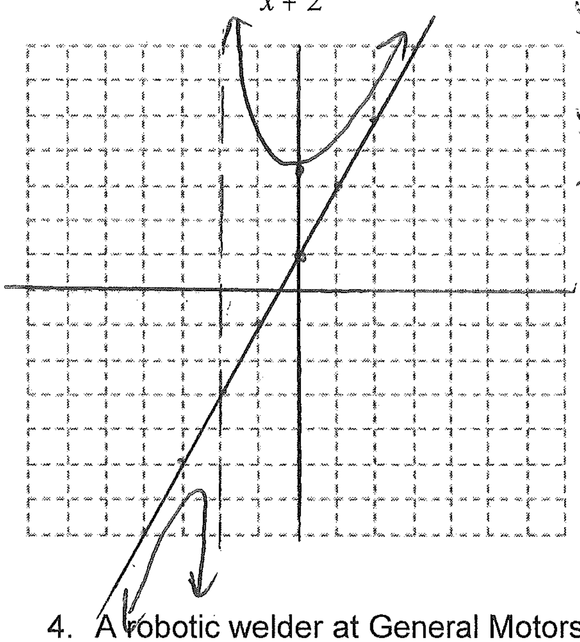
$y = x+5+\frac{16}{x-5}$

3. Find the linear oblique asymptote of each curve and use it to help you sketch the graph. (a) $y = 2x+1+\dfrac{3}{x+2}$ (b) $y = \dfrac{x^2-9}{x-5}$



$x=0$
$y=2.5$

$y=0$:

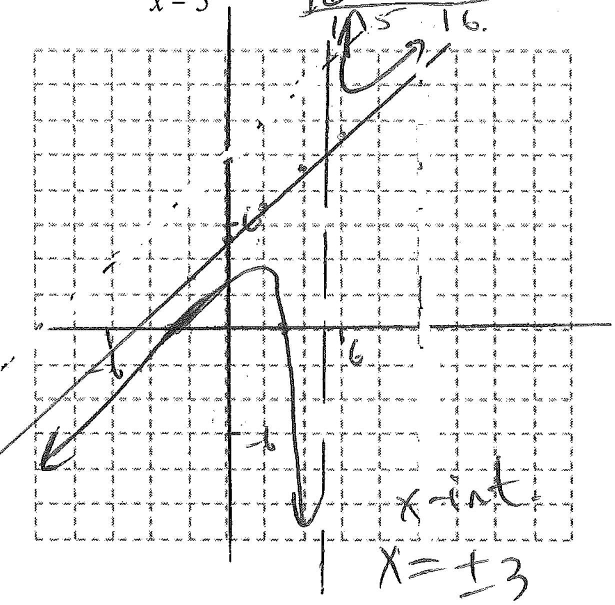
$2x+1 = \frac{-3}{x+2}$

$2x^2+5x+2=-3$

$2x^2+5x+5=0$

$x = \frac{-5 \pm \sqrt{25-4(2)(5)}}{10}$

No sol^n.



$$\begin{array}{r|rrr} 5 & 1 & 0 & -9 \\ & & 5 & 25 \\ \hline & 1 & 5 & 16. \end{array}$$

x-int:
$x = \pm 3$

4. A robotic welder at General Motors depreciates in value, D, in dollars, over time, t, in months. The value is given by $D(t) = 10000 - \dfrac{4000t}{(t+2)}$.

(a) Find the value of the machinery after:
(i) 1 month (ii) 6 months (iii) 1 year (iv) 10 years

(b) Would you have a local maximum or local minimum in the interval [0,4]? Explain.

(c) Find the D(t) is t becomes extremely large.

(d) Will the machinery ever have a value of $0?

(e) In light of your result in part (d), does V(t) model the value of the machinery for all time?

a) i) $D(1) = 10000 - \frac{4000}{3} = \$8667.67$ ii) $D(6) = \$7000$, $D(12) = \$6571.43$.

b) max at $t=0$. As $t \uparrow$ $D(t) \downarrow$

c) $D(t) = \$6000$ since $\frac{4000t}{t+2}$ will approach $\$4000$ as $t \to \infty$

d) NO. $6000 will be the minimum.

e) No as eventually, it will lose its worth (value will not stay at $6000).

*Answers on page 3

5. For the function $f(x) = \dfrac{x^2 + 5x + 6}{x - 2}$, use the domain, intercepts and vertical, horizontal and oblique asymptotes to sketch the graph.

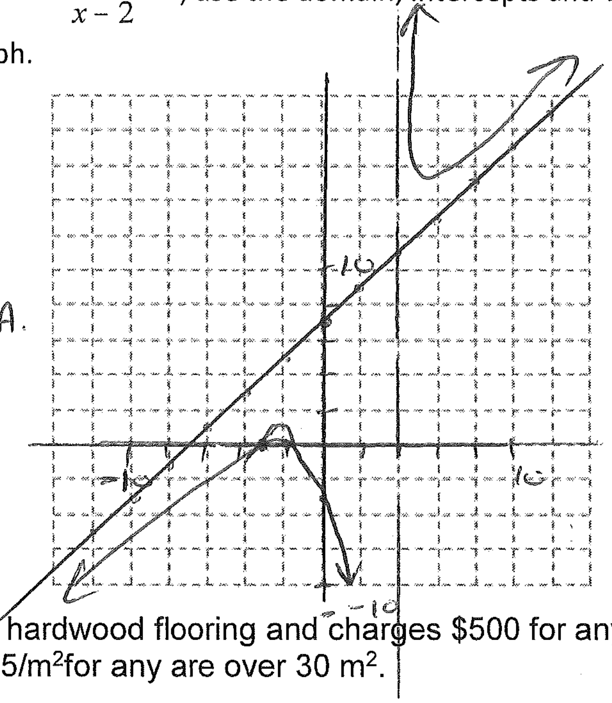
$$\begin{array}{r|rrr} 2 & 1 & 5 & 6 \\ & \downarrow & 2 & 14 \\ \hline & 1 & 7 & 20 \end{array}$$

$\therefore y = x + 7$ is the obliqueA.

D: $\{x \in \mathbb{R} \mid x \neq 0\}$

VA: $X = 2$

HA: NONE



x-int: $x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -3, -2$

y-int: $y = -3$.

6. Empire Flooring installs hardwood flooring and charges \$500 for any area less than or equal to 30 m^2 and an additional \$25/$m^2$for any are over 30 m^2.

   (a) Find a piecewise function y = c(x) to represent the average cost, per square metre, to install s square metre of carpet.

   (b) Find the value of c(x) as x becomes extremely large

   (c) Graph y = c(x) for x>0

   (d) Would it be economical to have this company install hardwood for an area of 5 m^2 ? Explain.

a) $c(x) = \begin{cases} \dfrac{500}{x}, & x \leq 30 \\ \dfrac{500}{x} + 25 & x > 30 \end{cases}$

b) $c(x) \to 25$

c).

d) NO.

Answers:

1.	(a) y = 2	(b)	y = 0	(c)	y = 1	(d)	y = 1
2.	(a) y = 2x-4	(b)	y=x	(c)	y = 3x+1	(d)y = x + 6	

3. (a) y = 2x + 1 (b) y = x + 5

4. (a) (i) \$8667.67 (ii) 7000 (iii) \$6571.43 (iv) 6065.57

   (b) max at t = 0

   (c) 6000 (d) no (e) no

5. Graph

6. (a) $C(x) = \dfrac{500}{x} x \leq 30$, $C(x) = \dfrac{500 + 25x}{x}, x > 30$ b) ~~525~~ 25 d) No

Graphing Rational Functions and Solving Inequalities

Answer questions 1 to 5 without graphing technology.
For question 1 – 6, refer to the following functions.

(a) $f(x) = \dfrac{x}{2x-10}$ (b) $g(x) = \dfrac{3x^2-5x+2}{0x^2+x+0}$

$y = 3x-5$ oblique.

1. What are the x- and y-intercepts of each function?

a) x-int: 0

y-int: 0

b) x-int: $(3x-2)(x-1)=0$

$x = 1, 2/3$

y-int: N/A.

2. Write the domain for each function.

a) $\{x \in \mathbb{R} \mid x \neq 5\}$

b) $\{x \in \mathbb{R} \mid x \neq 0\}$

3. What are the vertical asymptote(s)?

a) VA: $x=5$

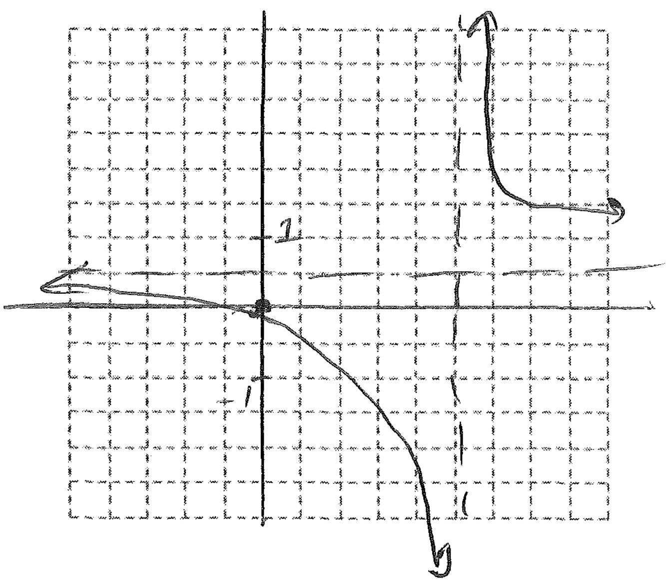
b) $x=0$

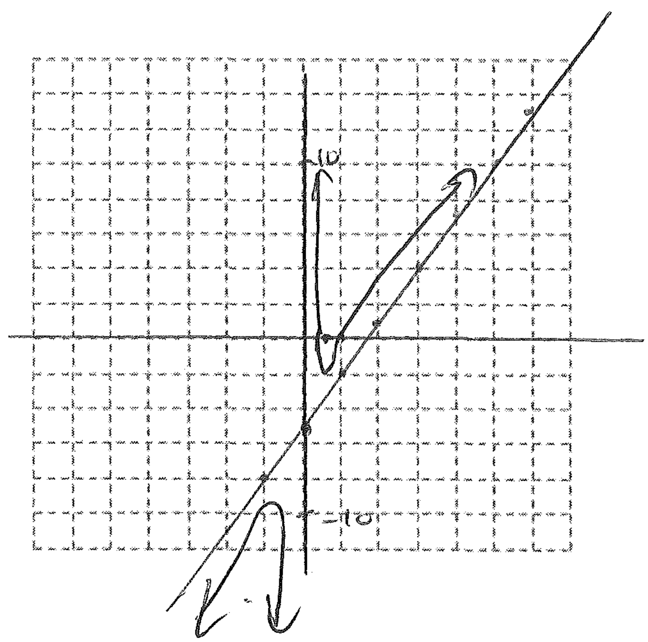
4. What are the horizontal asymptote(s)?

a) HA: $y = 1/2$

b) NONE

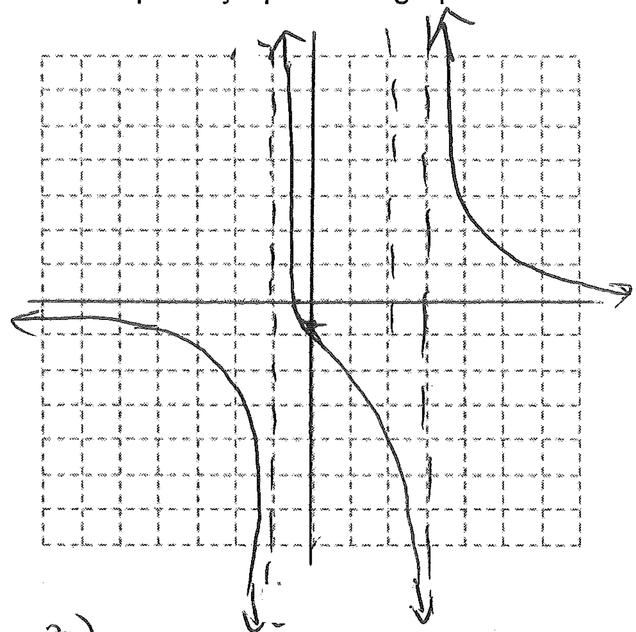
5. Use the information from questions 1 to 5 to graph each function.





6. Use the domain and intercepts, and vertical, horizontal and oblique asymptotes to graph each function.

(a) $f(x) = \dfrac{2x+1}{x^2-2x-3} = \dfrac{2x+1}{(x-3)(x+1)}$



VA: $x=3,-1$

HA: $y=0$

x-int: $-\frac{1}{2}$

y-int: $-1/3$

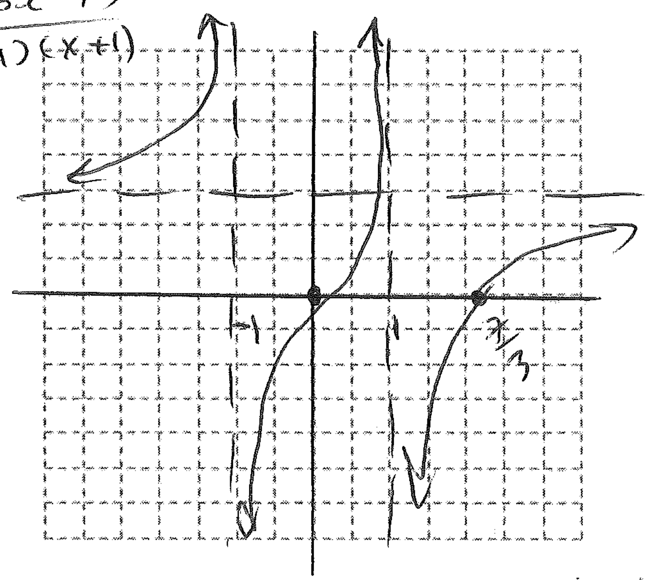
$x \to 3^+ \quad y \to \infty$

$x \to 3^- \quad y \to -\infty$

$x \to -1^+ \quad y \to \infty$

$x \to -1^- \quad y \to -\infty.$

(b) $g(x) = \dfrac{3x^2-7x}{x^2-1} = \dfrac{3x^2-7x}{(x-1)(x+1)} = \dfrac{x(3x-7)}{(x-1)(x+1)}$



x-int: $x=0, 7/3$

y-int: 0

VA: $x=\pm 1$

HA: $y=3$

$x \to 1^+ \quad y \to -\infty$

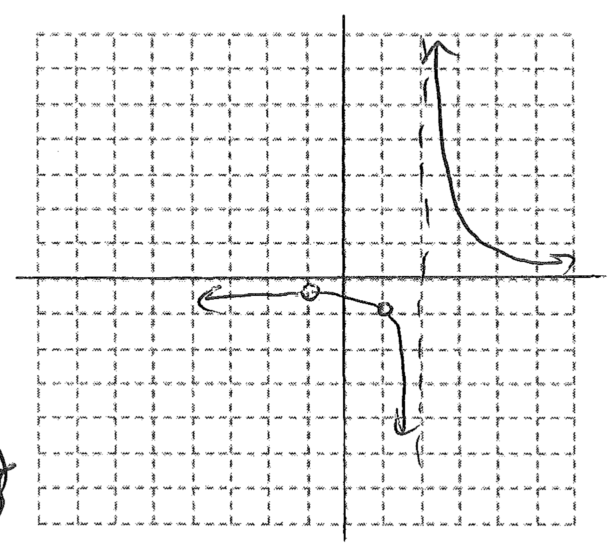
$x \to 1^- \quad y \to +\infty$

$x \to -1^+ \quad y \to -\infty$

$x \to -1^- \quad y \to \infty$

(c) $h(x) = \dfrac{x^2-1}{x^3-2x^2-x+2} = \dfrac{\cancel{(x-1)}\cancel{(x+1)}}{\cancel{(x-1)}\cancel{(x+1)}(x-2)} = \dfrac{1}{x-2}$

Hole at $x = 1, -1$
$(1,-1)$ $(-1,-1/3)$



x-int: NONE

y-int: $y = -1/2$

VA: $x=2$

HA: $y=0$

D: $\{x \in \mathbb{R} \mid x \neq 2, \pm 1\}$

*Answers on page 6 $\{y \in \mathbb{R} \mid y \neq 0, -1, -1/3\}$

7. Solve the following rational inequality.

$$\frac{x}{2x-8} \geq \frac{x^2+x-6}{x+2}$$

$$\frac{x}{2x-8} - \frac{x^2+x-6}{x+2} \geqslant 0$$

$$\frac{x(x+2)-(2x-8)(x^2+x-6)}{2(x-4)(x+2)} \geqslant 0$$

$$\frac{x^2+2x-2x^3-2x^2+12x+8x^2+8x-48}{2(x-4)(x+2)} \geqslant 0$$

will need technology.

8. What is a rational inequality? How do you solve a rational inequality?

An expression involving rational expressions and one symbol from $<$, $\leq$, $\geq$ or $>$.

To solve: work with 0 on one side.

10. For each case, create a function that has a graph with the given features.

(a) a vertical asymptote x = 3, a horizontal asymptote y =0, no x-intercepts, and y-intercept = -1

$$y = \frac{3}{x-3}$$

$$\frac{ax+b}{cx+d}$$

$a=0$

$d=-3$

$\frac{b}{d} = -1$

$\Rightarrow b = 3$

(b) a vertical asymptote the y-axis, an oblique asymptote y = 2x+1 and no x- and y-intercepts.

$$y = (2x+1) + \frac{1}{x} = \frac{x(2x+1)+1}{x} = \frac{2x^2+x+1}{x}$$

Answers:

1.	(a) x-int=0,y-int=0	(b) x-int=2/3,1, y-int=none
2.	(a) $x \neq 5$	(b) $x \neq 0$
3.	(a) x = 5	(b) x = 0
4.	(a) y = 0	(b) no
8.	x#-2.95, -2<x#1.71, 4<x#4.75	
10.	(a) $y = \frac{3}{x-3}$	(b) $y = \frac{2x^2+x+1}{x}$