Review for MCV4U-Unit 2

$f(-2) = \frac{2}{-8} = \frac{-1}{4}$

1. Determine $f'(-2)$ for the function $f(x) = \frac{2}{x^3}$ then find the equation of the tangent.

$f(x) = 2x^{-3}$

$f'(x) = -6x^{-4}$

$= \frac{-6}{x^4}$

$f'(-2) = \frac{-6}{(-2)^4} = \frac{-6}{16} = \frac{-3}{8}$

$y - y_1 = m(x - x_1)$

$y + \frac{1}{4} = -\frac{3}{8}(x+2)$

$y = -\frac{3}{8}x - \frac{6}{8} - \frac{1}{4} = \frac{-3}{8}x - 1$

2. Differentiate $h(x) = (\sqrt{x}+1)(3x^2 - 5x)$ and leave the answer unsimplified.

$h'(x) = \left(\frac{1}{2\sqrt{x}}\right)(3x^2 - 5x) + (\sqrt{x}+1)(6x-5)$

3. Differentiate $f(x) = x(4x-1)(6x+3)$

$f'(x) = 1(4x-1)(6x+3) + (x)(4)(6x+3) + (x)(4x-1)(6)$

4. Find the derivative of the following:

a) $f(x) = (x^2 - x + 2)^8$

$f'(x) = 8(x^2 - x + 2)^7(2x-1)$

b) $f(x) = \sqrt{2x^2+3}$

$= (2x^2+3)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(2x^2+3)^{\frac{-1}{2}}(4x)$

$= (2x)(2x^2+3)^{\frac{-1}{2}}$

c) $f(x) = \frac{1}{\sqrt[3]{1-x^4}} = (1-x^4)^{\frac{-1}{3}}$

$f'(x) = -\frac{1}{3}(1-x^4)^{\frac{-4}{3}}(-4x^3)$

$= +\frac{4x^3}{3}(1-x^4)^{\frac{-4}{3}}$

A. Determine where the function $f(x) = \sqrt{2x^2 - x - 3}$ is NOT differentiable.

$f(x) = (2x^2 - x - 3)^{\frac{1}{2}}$

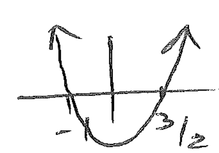
$f'(x) = \frac{1}{2}(2x^2 - x - 3)^{\frac{-1}{2}}(4x-1)$

$= \frac{4x-1}{2\sqrt{2x^2-x-3}}$

$2x^2 - x - 3 > 0$

$x \in (-\infty, -1) \cup (\frac{3}{2}, \infty)$

$\sqrt{(2x-3)(x+1)}$



$x \geq \frac{3}{2},$

$x \leq -1$

5. Differentiate $f(x) = (x^2+1)^3(2-3x)^4$. Your final answer should be fully factored.

$$f'(x) = 3(x^2+1)^2(2x)(2-3x)^4 + (x^2+1)^3(4)(2-3x)^3(-3)$$

$$= 6x(x^2+1)^2(2-3x)^4 - 12(x^2+1)^3(2-3x)^3.$$

$$= 6(x^2+1)^2(2-3x)^3[x(2-3x) - 2(x^2+1)]$$

$$= 6(x^2+1)^2(2-3x)^3(2x - 3x^2 - 2x^2 - 2)$$

$$= 6(x^2+1)^2(2-3x)^3(-5x^2+2x-2)$$

6. Determine the equation of the tangent to $y = \frac{2x}{x^2+1}$ at $x = 0$.

$$y' = \frac{2(x^2+1) - (2x)(2x)}{(x^2+1)^2}$$

$$m = y'(0) = \frac{2(0^2+1) - 2(0)(2(0))}{(0^2+1)^2}$$

$$= 2$$

$y(0) = 0$

$\therefore y = 2x$ is the eq^n.

since y-intercept is 0.

OR $y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 0)$

$y = 2x$.

7. If $y = 2u^4 - 3u^2 + u - 1$, and if $u = 4\sqrt{x}$, then evaluate $\frac{dy}{dx}$ at $x = 1$

$y' = (8u^3 - 6u + 1)\left(4 \cdot \frac{1}{2\sqrt{x}}\right)$ when $x = 1$, $u = 4$

$$y'(x=1) = [8(4)^3 - 6(4) + 1]\left[\frac{4}{2\sqrt{1}}\right]$$

$$= (489)(2) = 978$$

8. If $f(x)$ is a differentiable function, determine an expression for the derivative of $g(x) = 3x^2 f(x^2 - 3)$.

Product rule.

$$g'(x) = (6x)(f(x^2-3)) + (3x^2) f'(x^2-3) \cdot 2x$$

$$= 6x[f(x^2-3) + x^2 f'(x^2-3)]$$

Challenge:

9. What is the maximum slope of a tangent to the function $f(x) = -x^3 + 4x - 2$?

$f'(x) = -3x^2 + 4 \rightarrow$ This is a slope function. It is a parabola. The maximum is 4 when $x = 0$.

10. Show that there are no tangent to the curve $f(x) = \frac{5x+2}{x+2}$ that have a negative slope. $, x \neq -2$

We need to show $f'(x) > 0$ for any x.

$$f'(x) = \frac{5(x+2) - (5x+2)(1)}{(x+2)^2} = \frac{5x + 10 - 5x - 2}{(x+2)^2} = \frac{8}{(x+2)^2}$$

Since $f'(x) = \frac{8}{(x+2)^2}$, $8 > 0$ and $(x+2)^2 > 0$ $(x \neq -2)$

$\therefore$ $f'(x)$ is always positive.

11. Determine the derivative of $k(x) = \left(\frac{x^3 - 3}{x^3 + 3}\right)^8$ using two different methods.

$$k(x) = \frac{(x^3-3)^8}{(x^3+3)^8}$$

$$k'(x) = \frac{8(x^3-3)^7(3x^2)(x^3+3)^8 - (x^3-3)^8(8)(x^3+3)^7(3x^2)}{(x^3+3)^{16}}$$

$$= \frac{24x^2(x^2-3)^7(x^3+3)^7[x^3+3-(x^3-3)]}{(x^3+3)^{16}}$$

$$= \frac{24x^2(x^2-3)^7(6)}{(x^3+3)^9}$$

$$= \frac{144x^2(x^2-3)^7}{(x^3+3)^9}$$

$$k(x) = [g(x)]^8$$

$$k'(x) = 8(g(x))^7 \cdot g'(x)$$

$$\therefore k'(x) = 8\left(\frac{x^3-3}{x^3+3}\right)^7\left(\frac{3x^2(x^3+3) - 3x^2(x^3-3)}{(x^3+3)^2}\right)$$

$$= \frac{8(x^3-3)^7(3x^2(6))}{(x^3+3)^7(x^3+3)^2}$$

$$= \frac{144x^2(x^3-3)^7}{(x^3+3)^9}$$

12. The normal to $f(x) = \frac{1}{x-1} = (x-1)^{-1}$ at (2, 1) intersects the graph of $f(x)$ at another point. What are the coordinates of the other point?

$f'(x) = (-1)(x-1)^{-2}$

$= \frac{-1}{(x-1)^2}$

$f'(2) = -1 \Rightarrow$ normal $= 1$

$\therefore y - y_1 = m(x - x_1)$

$y - 1 = 1(x-2)$

$\boxed{y = x - 1}$

$\frac{1}{x-1} = x - 1$

$\therefore (x-1)^2 = 1$

$x - 1 = \pm 1$

$x = 2$ or $x = 0 \rightarrow y = \frac{1}{0-1}$

$\therefore$ the coordinates are $(0, -1)$

13. Determine the derivative of $f(x) = (x+1)^2(3x-5)^4$. Write your answer in simplified factored form.

$f'(x) = 2(x+1)(3x-5)^4 + (x+1)^2(4)(3x-5)^3(3)$

$= 2(x+1)(3x-5)^4 + 12(x+1)^2(3x-5)^3$

$= 2(x+1)(3x-5)^3[3x - 5 + 6(x+1)]$

$= 2(x+1)(3x-5)^3(9x+1)$

b. Determine the value(s) of x for which the graph of $f(x)$ has a horizontal tangent.

horizontal tangent $\Rightarrow f'(x) = 0$

$\therefore x = -1, \frac{5}{3}, \frac{-1}{9}.$