Chapter 1 PRE-TEST REVIEW – Polynomial Functions
MHF4U

Section 1: 1.1 Power Functions

1) State the degree and the leading coefficient of each polynomial

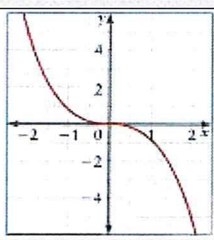
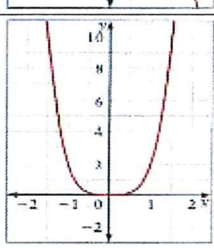
Polynomial	Degree	Leading Coefficient
$y = 2x^3 + 3x - 1$	3	2
$y = 5x - 6$	1	5
$y = x^3 - 2x^2 - 5x^4 + 3$	4	−5
$y = -3x^5 + 2x^3 - x - 1$	5	−3
$y = 21 - 2x + 4x^2 - 6x^3$	3	−6

2) Match each function to its end behavior

$y = 3x^7$ $\quad$ $y = -\frac{1}{2}x^3$ $\quad$ $y = 2x^4$ $\quad$ $y = -0.25x^6$

End Behaviour	Functions
Q3 to Q1	$y = 3x^7$
Q2 to Q4	$y = -\frac{1}{2}x^3$
Q2 to Q1	$y = 2x^4$
Q3 to Q4	$y = -0.25x^6$

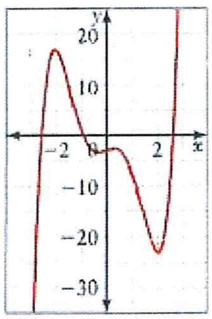
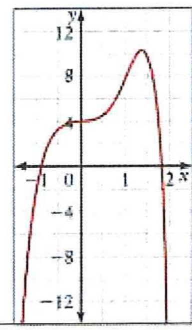
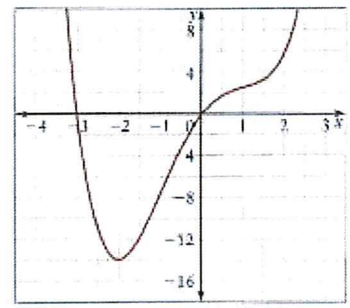
3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	odd	neg	$x \in (-\infty, \infty)$ $y \in (-\infty, \infty)$	odd	Q2 → Q4
	even	positive	$x \in (-\infty, \infty)$ $y \in [0, \infty)$	even	Q1 → Q2

Section 2: 1.2 Characteristics of Polynomial Functions

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

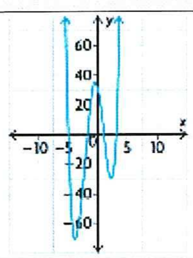
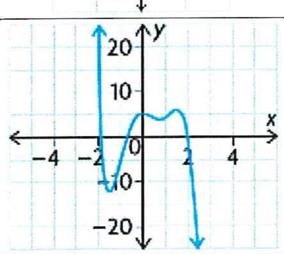
A) $g(x) = 0.5x^4 - 3x^2 + 5x$ **B)** $h(x) = x^5 - 7x^3 + 2x - 3$ **C)** $p(x) = -x^6 + 5x^3 + 4$

		
B	C	A

5) Complete the following table

Equation	Degree	Sign of Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = 6x^3 + 2x$	3	+	Q1→Q3	0, 2	1, 2, 3
$g(x) = -20x^6 - 5x^3 + x^2 - 17$	6	−	Q3→Q4	1,3,5	0 to 6
$p(x) = 22x^4 - 4x^3 + 3x^2 - 2x + 2$	4	+	Q1→Q2	1, 3	0 to 4
$h(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$	5	−	Q2→Q4	0,2,4	1 to 5

6) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q1→Q2	neither	3	4	4
	-	odd	Q2→Q4	Neither	4	3	5

7) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) fifth differences = -60

$a(5\times4\times3\times2\times1) = -60$

$120a = -60$

$a = -\frac{1}{2}$

b) third differences = 42

$a(3\times2\times1) = 42$

$6a = 42$

$a = 7$

8) For each function, find the value of the constant finite differences.

a) $g(x) = 0.5x^4 - 3x^2 + 5x$

$a(4!) =$ 4th diff.

$0.5(4\times3\times2\times1) =$ 4th diff

4th diff $= 12$

b) $h(x) = x^5 - 7x^3 + 2x - 3$

$5!\,a =$ 5th diff.

5th differ $= (5\times4\times3\times2\times1)(1)$

$= 120$

9) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
−3	124
−2	41
−1	8
0	1
1	−4
2	−31
3	−104
4	−247

41-124

-83, -33, -7, -5, -27, -73, -143

-247-(-104)

50, 26, 2, -22, -46, -70

-24, -24, -24, -24, -24

$\therefore$ degree $= 3$

$6a = -24$

$\boxed{a = -4}$ is the leading coefficient.

b)

x	y
−2	−229
−1	−5
0	3
1	−7
2	−53
3	−129
4	35
5	1213

+224, 8, -10, -46, -76, 164, 1178

-216, -18, -36, -30, 240, 1014

198, -18, 6, 270, 774

-216, 24, 264, 504

240, 240, 240

5th differences constant implies degree 5.

$5!\,a = 240$

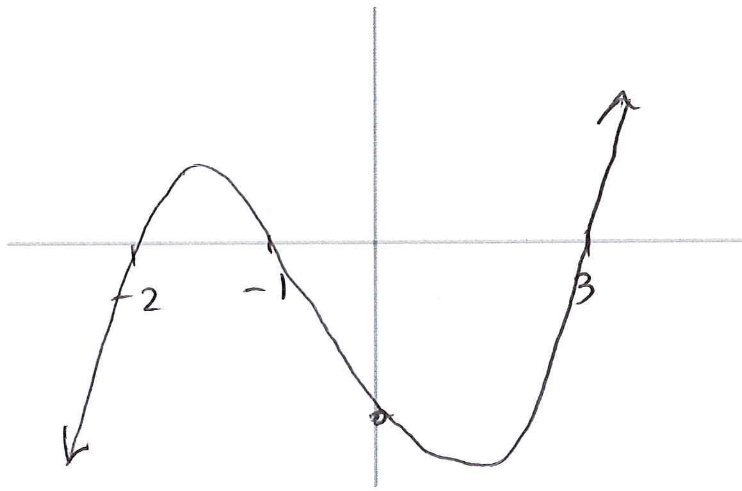
$\boxed{a = 2}$ is the LC

Section 3: 1.3 Factored Form Polynomial Functions

10) For each function, complete the chart and sketch a possible graph of the function labelling key points.

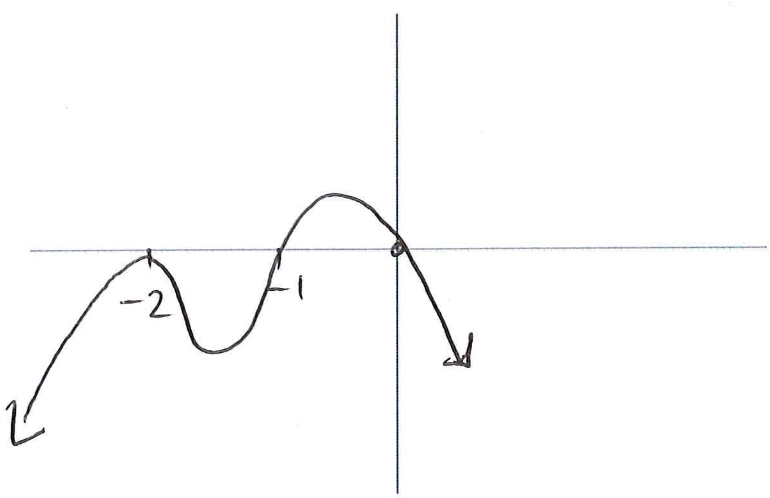
a) $f(x) = (x+1)(x-3)(x+2)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	+ (1)	Q1 → Q3	$x=-1$ $x=3$ $x=-2$	$x=0$ $y=-6$



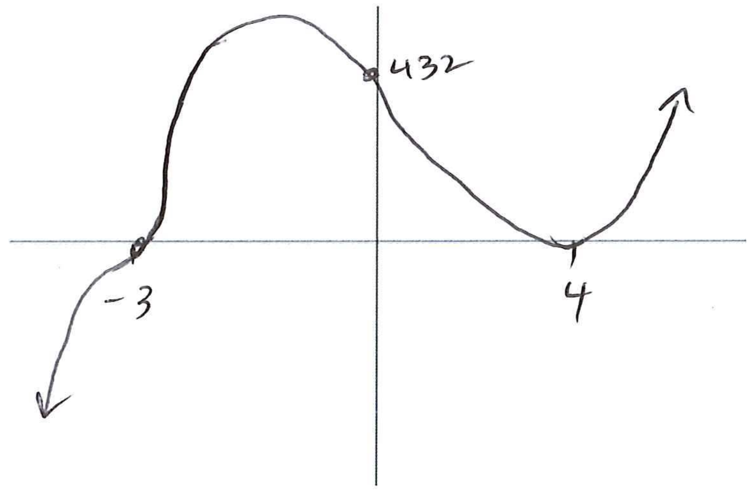
b) $g(x) = -x(x+1)(x+2)^2$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
4	− (-1)	Q3 → Q4	$x=0$ $x=-1$ $x=-2$ (order 2)	0



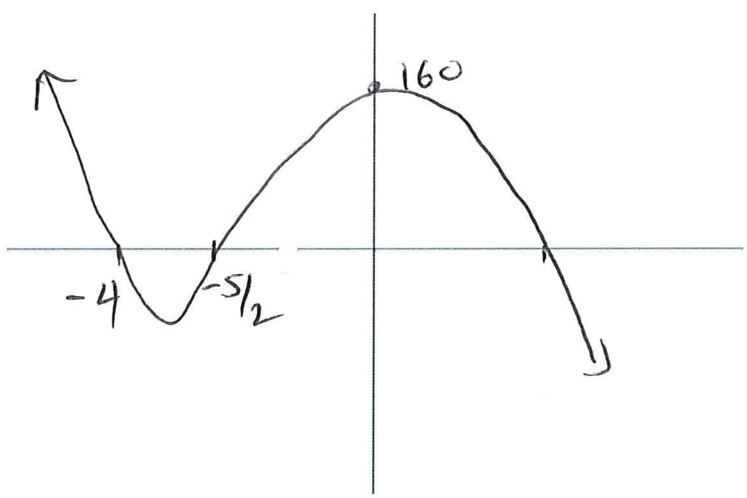
c) $h(x) = (x-4)^2(x+3)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
5	+ (1)	Q1→Q3	x=4 (order 2) x=-3 (order 3)	x=0 y= 432



d) $p(x) = -4(2x+5)(x-2)(x+4)$

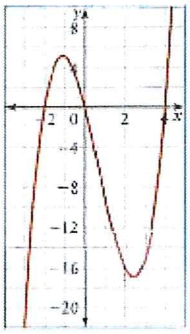
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	− a=-8	Q2→Q4	x=-5/2 x=2 x=-4	160



11) For each graph, state...

- **i)** the least possible degree and the sign of the leading coefficient
- **ii)** the x-intercepts (specify order of zero) and the factors of the function
- **iii)** the intervals where the function is positive/negative

a)



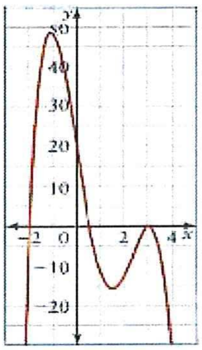
i) degree: 3
leading coefficient: +

ii) x-intercepts: $x = 0, -2, 4$
factors: $(x)(x+2)(x-4)$

iii)

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,4)$	$(4,\infty)$
Sign	−	+	−	+

b)



i) degree: 4
leading coefficient: −

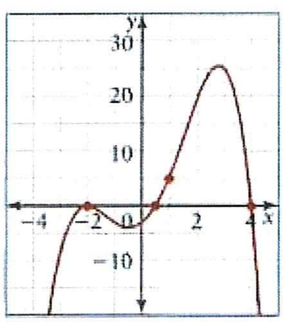
ii) x-intercepts: $x = -2, \frac{1}{2}, 3$ (order 2)
factors: $(x+2)(2x-1)(x-3)^2$

iii)

Interval	$(-\infty,-2)$	$(-2,1/2)$	$(1/2,3)$	$(3,\infty)$
Sign	−	+	−	−

12) Write the equation of each of the following functions:

a)



$y = a(x+2)^2(2x-1)(x-4)$

sub $x=1$ $y=5$

$5 = a(1+2)^2(2-1)(1-4)$

$5 = a(9)(1)(-3) \Rightarrow 5 = -27a$

$a = \frac{-5}{27}$

$\therefore y = \frac{-5}{27}(x+2)^2(2x-1)(x-4)$

b) The quartic function has zeros at -3, -1, and 2 (order 2) and passes through the point (1, 4)

$y = a(x+3)(x+1)(x-2)^2$

$4 = a(4)(2)(1)$

$a = 1/2 \Rightarrow y = \frac{1}{2}(x+3)(x+1)(x-2)^2$

Section 4: 1.4 Transformations of Polynomial Functions

12) Write an equation for the function that results from the given transformations.

a) The function $f(x) = x^4$ is compressed vertically by a factor of $\frac{3}{5}$, stretched horizontally by a factor of 2, reflected horizontally in the y-axis, and translated 1 unit up and 4 units to the left.

$f(x) = \frac{3}{5}\left(-\frac{1}{2}(x+4)\right)^4 + 1$

b) The function $f(x) = x^3$ is compressed horizontally by a factor of $\frac{1}{4}$, stretched vertically by a factor of 5, reflected vertically in the x-axis, and translated 2 units to the left and 7 units up.

$f(x) = -5\left(4(x+2)\right)^3 + 7$

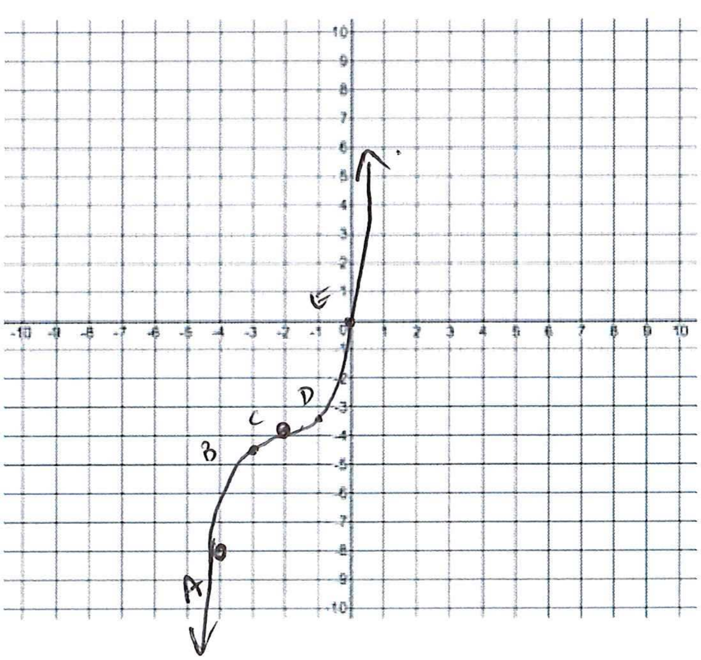
13) Identify the a, k, d and c values and explain what transformation is occurring to the parent function for $g(x) = 2[-4(x+7)]^4 - 1$ $a = 2$ $k = -4$ $d = -7$ $c = -1$

- reflection in y-axis
- vertically stretched by a factor of 2
- horizontally compressed by a factor of 1/4
- 7 units to the left and 1 unit down.

14) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^3$ $g(x) = \frac{1}{2}f(x+2) - 4$

$(x,y) \to (x-2, \frac{1}{2}y-4)$



$(-2,-8) \to (-4,-8)$ A

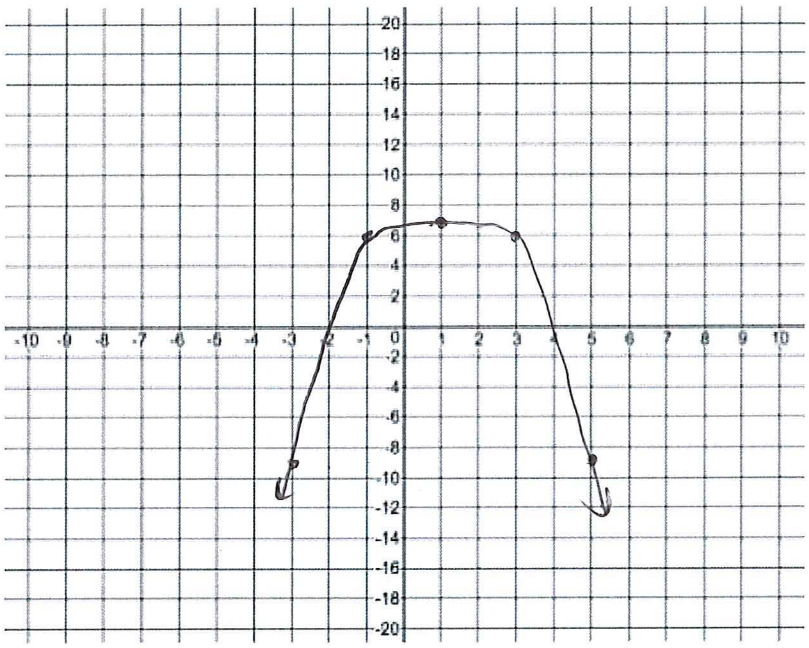
$(-1,-1) \to (-3,-4.5)$ B

$(0,0) \to (-2,-4)$ C

$(1,1) \to (-1,-3.5)$ D

$(2,8) \to (0,0)$ E

b) $f(x) = x^4$ $\qquad$ $g(x) = -f\left[\frac{1}{2}(x-1)\right] + 7$



$(x,y) \to (2x+1, -y+7)$

$(2,16) \to (5, -9)$ A

$(1,1) \to (3, 6)$ B

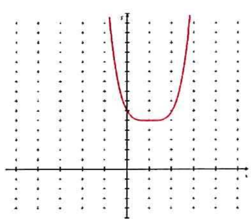
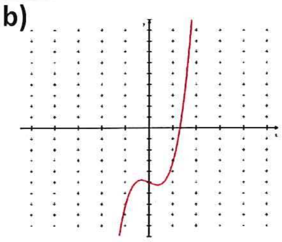
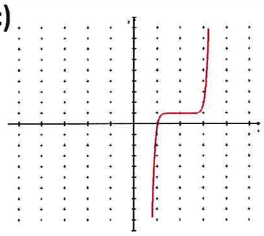
$(0,0) \to (1, 7)$ C

$(-1,1) \to (-1, 6)$ D

$(-2, 16) \to (-3, -9)$ E.

Section 5: 1.5 Symmetry

15) Circle all that apply for each function

a) 	No symmetry Even function Odd function Line Symmetry Point Symmetry
b) 	No symmetry Even function Odd function Line Symmetry Point Symmetry
c) 	No symmetry Even function Odd function Line Symmetry Point Symmetry

d) $f(x) = 3x^6 + 2x^2 - 5$	No symmetry Even function Odd function Line Symmetry Point Symmetry
e) $f(x) = x^3 - 4x^2 + 1$	No symmetry Even function Odd function Line Symmetry Point Symmetry
f) $f(x) = x^4 + 5x$	No symmetry Even function Odd function Line Symmetry Point Symmetry

16) Consider the polynomial function $f(x) = -3x^4 + 6x^2 - 10$

a) Show algebraically whether f is even, odd or neither.

$f(-x) = -3(-x)^4 + 6(-x)^2 - 10$

$= -3x^4 + 6x^2 - 10$

Since $f(x) = f(-x) \Rightarrow f(x)$ is EVEN.

b) For what finite difference will f give a constant value, and what will that constant value be?

4th differences will be constant.

$a(4!) =$ 4th diff

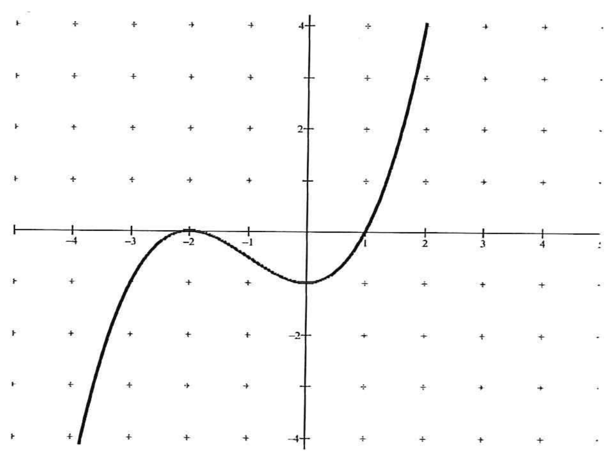
$-3(24) =$ 4th diff. $\therefore$ 4th differences will be -72.

c) What are the maximum and minimum number of zeros the above polynomial could have?

Minimum number of zeros will be 0.

Maximum number of zeros will be 4.

17) Use the given graph to state:



a) x-intercepts $x = -2$ (order 2)

$x = 1$

b) number of turning points 2

c) least possible degree 3

d) any symmetry present; even or odd function?

NEITHER

e) the intervals where $f(x) < 0$ $x \in (-\infty, -2) \cup (-2, 1)$