Name: SOLUTIONS

Date:______________________

Unit 2 – Rational Expressions REVIEW

PART 1: KNOWLEDGE AND UNDERSTANDING

1. The correct product of $\frac{(a-3)\cancel{(a-2)}}{(a-1)\cancel{(a+1)}} \times \frac{(a-5)\cancel{(a+1)}}{\cancel{(a-2)}(a+2)}$ simplifies fully to the form:

$\frac{(a-3)(a-5)}{(a-1)(a+2)}$

a) $\frac{(a+3)(a-5)}{(a-1)(a+2)}$ b) $\frac{(a-3)(a-5)}{(a-1)(a-2)}$

c) $\frac{(a-3)(a+5)}{(a+1)(a+2)}$ d) $\frac{(a-3)(a-5)}{(a-1)(a+2)}$

2. Which expression has restrictions $m \neq -2,\ m \neq 2$?

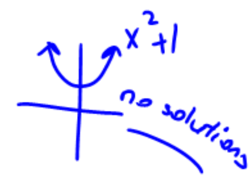
$m+2 \neq 0$ $m-2 \neq 0$

$(m+2)(m-2)$

a) $\frac{3m}{m-2} \times \frac{4(m-2)}{6m}$ b) $\frac{10m}{m-2} \div \frac{5}{2(m+2)}$

c) $\frac{(3m+1)}{(2m-1)} \times \frac{2m-1}{3m(m+1)}$ d) $\frac{5m(m+3)}{4m} \times \frac{(m-5)}{(m^2-2)}$

3. Which rational expression has no restrictions?



a) $\frac{x^2+6x+9}{4x}$ b) $\frac{4}{(x+3)^2}$ c) $\frac{9x^2}{-9x^2}$ d) $\frac{x^2}{1+x^2}$

$4x \neq 0$, $x \neq 0$

$x+3 \neq 0$, $x \neq -3$

$-9x^2 \neq 0$, $x \neq 0$

$1+x^2 \neq 0$, $\sqrt{x^2} \neq \sqrt{-1}$, $x \neq$ undefined

4. Given the rational expression $\frac{x+5}{3x^2+6x}$ the restrictions on the variable are:

$\frac{(x+5)}{3x(x+2)}$

$x \neq 0$ $x \neq -2$

a) $x \neq -3$ and $x \neq 0$ b) $x \neq -2$ and $x \neq 2$

c) $x \neq -2$ and $x \neq 0$ d) $x \neq 0$

5. The lowest common denominator required to perform the operation $\frac{3x}{x-3} + \frac{4x-1}{x^2-5x+6}$ is:

a) $(x-2)^2(x-3)$ b) $(x-2)(x-3)$

c) $(x+2)(x-3)$ d) $(x^2-5x+6)(x-2)$

$\frac{3x}{(x-3)} + \frac{4x-1}{(x-2)(x-3)}$

LCD $(x-2)(x-3)$

6. The rational expression $\frac{x+1}{x^2-1}$ can be reduced to the form:

a) $\frac{-1}{x-1}$ b) $x-1$ c) $\frac{(1+x)}{(x-1)(x+1)}$ d) $\frac{1}{x-1}$

$\frac{\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{1}{x-1}$

MCR3U1 Name:______________________________

$$\frac{1-3x}{x^2+9} \div \frac{2}{(2-x)(2+x)} = \frac{1-3x}{x^2+9}\cdot\frac{(2-x)(2+x)}{2}$$

no rest

7. The quotient $\frac{1-3x}{x^2+9} \div \frac{2}{4-x^2}$ has a total of: $\boxed{x \neq \pm 2}$

a) Two restrictions $\neq \pm 2$
b) Three restrictions
c) Four restrictions
d) Five restrictions

8. Simplify fully **and** state restrictions. You must show how you:

- Factored fully including: common factors, simple trinomials, decomposition, difference of squares
- Reduced each rational expression to lowest terms (where applicable)
- Simplified fully (either by reducing, multiplying/dividing, or adding/subtracting)

a) $\frac{x^2-1}{2x^2-3x+1}$

M: 2
A: −3
N: −1 −2

$$\frac{(2x-1)(2x-2)}{2} \rightarrow \frac{(2x-1)(2)(x-1)}{\cancel{2}}$$

$$= \frac{\cancel{(x-1)}(x+1)}{(2x-1)\cancel{(x-1)}} \Rightarrow x \neq \frac{1}{2},\ x \neq 1$$

$$= \frac{x+1}{2x-1} \Rightarrow x \neq \frac{1}{2},\ x \neq 1$$

b) $\frac{2}{(x-3)} - \frac{x}{x^2-8x+15}$

$$= \frac{2}{(x-3)} - \frac{x}{(x-3)(x-5)}$$

LCD $(x-3)(x-5)$

$x \neq 3, 5$

$$= \frac{2(x-5)}{(x-3)(x-5)} - \frac{x}{(x-3)(x-5)}$$

$$= \frac{2x-10-x}{(x-3)(x-5)} \Rightarrow \frac{x-10}{(x-3)(x-5)} \quad x \neq 3, 5$$

c) $\frac{x^2-9x+14}{x^2+7x+12} \div \frac{x(7-x)}{4(x+4)}$

$$= \frac{(x-2)(x-7)}{(x+3)(x+4)} \div \frac{x(7-x)}{4(x+4)} \Rightarrow x \neq -3,\ x \neq -4$$

$$= \frac{(x-2)\cancel{(x-7)}^{-1}}{(x+3)\cancel{(x+4)}} \cdot \frac{4\cancel{(x+4)}}{x\cancel{(7-x)}} \Rightarrow x \neq -3,\ x \neq -4,\ x \neq 0,\ x \neq 7$$

$$= \frac{-4(x-2)}{x(x+3)} \Rightarrow x \neq -4, -3, 0, 7$$

d) $\frac{x^2}{x+2} + \left[\frac{x^2+x-12}{x^2+2x} \div \frac{(2x+1)(x-3)}{4x^3+2x^2}\right]$

$$\frac{x^2}{x+2} + \left[\frac{(x-3)(x+4)}{x(x+2)} \div \frac{(2x+1)(x-3)}{2x^2(2x+1)}\right] \Rightarrow x \neq -\frac{1}{2},\ x \neq 0,\ x \neq -2$$

$$\frac{x^2}{x+2} + \left[\frac{\cancel{(x-3)}(x+4)}{\cancel{x}(x+2)} \cdot \frac{2x^{\cancel{2}}\cancel{(2x+1)}}{\cancel{(2x+1)}\cancel{(x-3)}}\right] \Rightarrow x \neq -2, 0, -\frac{1}{2}, 3$$

$$\frac{x^2}{x+2} + \frac{2x(x+4)}{x+2} \Rightarrow x \neq -2$$

$$= \frac{x^2+2x^2+8x}{x+2} \Rightarrow \frac{3x^2+8x}{x+2} \quad x \neq -2, -\frac{1}{2}, 0, 3$$

9. In each of the examples below errors were made. Either the final answer is wrong, or the restriction is wrong, or both. Correct <u>all errors</u> for each rational expression.

a) $\frac{3-y}{y-3} = -1, \quad y \neq 0$

$y \neq 3$

b) $\frac{(y-4)^2}{y^2-16} = 1, \quad y \neq 4$ wrong

wrong

$$\frac{(y-4)\cancel{(y-4)}}{\cancel{(y-4)}(y+4)} = 1 \Rightarrow y \neq -4, +4$$

$$\frac{y-4}{y+4} \quad y \neq -4, 4$$

10. The length of a flag can be represented by the expression $9+3x$ and the area can be represented by the expression $3x^2+30x+63$ respectively.

a) Write a simplified expression to represent the width of the flag. State restrictions.
b) Find a simplified expression to represent the perimeter of the flag.
c) Do any restrictions on the variable apply? Justify.

$A = 3x^2+30x+63$

w

$9+3x$

a) $A = L \times w$

$(3x^2+30x+63) = (9+3x)w$

$\dfrac{\cancel{3}(x^2+10x+21)}{\cancel{3}(3+x)} = \dfrac{\cancel{3(3+x)}w}{\cancel{3(3+x)}}$ divide each side by 3(3+x)

$\dfrac{(\cancel{x+3})(x+7)}{(\cancel{x+3})} = w \Rightarrow x \neq -3$ factor wherever possible
note restrictions
simplify

$\boxed{w = x+7}$

b) $P = 2(L+w)$
$= 2(9+3x+x+7)$
$= 2(4x+16)$
$P = 8x+32$ Since the perimeter is a linear function there is no restriction.

11. Write a single rational expression with the two restrictions $x \neq 0$ and $x \neq -\frac{1}{2}$.

if $x \neq 0$, then x is being multiplied by any number.
if $x \neq \frac{-1}{2}$, then $x+\frac{1}{2} \neq 0$

$\therefore \dfrac{1}{x\left(x+\frac{1}{2}\right)}$

12. Solve for x if the reciprocal of $\left(\frac{1}{x}-1\right)$ is -2.

flip, multiply

① $\dfrac{1}{\frac{1}{x}-1} = -2 \Rightarrow$ ② $\dfrac{1}{\frac{1-x}{x}} = -2 \Rightarrow$ ③ $1 \div \dfrac{1-x}{x} = -2 \Rightarrow$ ④ $\dfrac{x}{1-x} = -2 \Rightarrow x \neq 1$

Same thing

$x \neq 0$

cross x

$\Downarrow$

$x = -2+2x$

$\boxed{2 = x}$, $x \neq 0, 1$

13. Simplify fully and state the restrictions. Show your factored steps, but not your factoring work.

a) $\dfrac{24xy^2z^3}{16x^3yz}$ $x \neq 0$, $y \neq 0$, $z \neq 0$

$= \dfrac{3yz^2}{2x^2}$ $x \neq 0$, $y \neq 0$, $z \neq 0$

b) $\dfrac{20mn}{24n^2} \times \dfrac{3n}{5m^2}$ $m \neq 0, n \neq 0$

$= \dfrac{(20)(3)mn^2}{(24)(5)m^2n^2}$

$= \dfrac{1}{2m}$ $m, n \neq 0$

c) $\dfrac{2a^2 + ab - 3b^2}{b^2 - a^2}$

M: $2 \times -3 = -6$
A: 1
N: $-2, +3$

$\dfrac{(2a-2b)(2a+3b)}{2}$

$\dfrac{2(a-b)(2a+3b)}{2}$

$= \dfrac{-1(a-b)(2a+3b)}{(b-a)(b+a)}$ $b \neq a$, $b \neq -a$

$= \dfrac{2a+3b}{a+b}$ $a \neq b$, $a \neq -b$

d) $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x^2}} = \dfrac{\frac{x+1}{x}}{\frac{x^2-1}{x}} = \underbrace{\dfrac{x+1}{x} \div \dfrac{x^2-1}{x}}$ $\Rightarrow x \neq 0$

$\dfrac{(x+1)}{x} \cdot \dfrac{x}{(x-1)(x+1)}$ $x \neq 0$, $x \neq \mp 1$

$= \dfrac{1}{x-1}$ $x \neq 0, x \neq \mp 1$

14. Simplify fully and state all restrictions.

$\dfrac{w^2 + 4w - 21}{w^2 + 2w - 35} \div \dfrac{9w^2 - 1}{3w^2 - 16w + 5}$

$\dfrac{(3w-1)(3w-15)}{3}$ M: $3 \times 5 = 15$, A: -16, N: $-1, -15$

$\dfrac{(3w-1)(3)(w-5)}{3}$

① Factor

$= \dfrac{(w-3)(w+7)}{(w-5)(w+7)} \div \dfrac{(3w-1)(3w+1)}{(3w-1)(w-5)}$ $w \neq 5, w \neq -7, w \neq \frac{1}{3}$

$= \dfrac{(w-3)(w+7)}{(w-5)(w+7)} \cdot \dfrac{(3w-1)(w-5)}{(3w-1)(3w+1)}$ $w \neq 5, w \neq -7, w \neq \mp\frac{1}{3}$

$= \dfrac{w-3}{3w+1}$ $w \neq -7, -\frac{1}{3}, \frac{1}{3}, 5$

15. Simplify and reduce fully, and state all restrictions.

$$\frac{y+4}{y^2+y-2}-\frac{6}{y^2-5y-14}$$

$$=\frac{(y+4)}{(y-1)(y+2)}-\frac{6}{(y+2)(y-7)}$$

$LCD=(y-1)(y+2)(y-7)$ $y\neq 1,-2,7$

$$=\frac{(y+4)(y-7)}{(y-1)(y+2)(y-7)}-\frac{6(y-1)}{(y+2)(y-7)(y-1)}$$

$$=\frac{y^2-7y+4y-28-6y+6}{(y-1)(y+2)(y-7)}$$

$$=\frac{y^2-9y-22}{(y-1)(y+2)(y-7)}$$

$\rightarrow (y+2)(y-11)$

$M=1\times -22=-22$
$A=-9$
$N=+2,\ -11$

$$=\frac{\cancel{(y+2)}(y-11)}{(y-1)\cancel{(y+2)}(y-7)}$$ $y\neq 1,-2,7$

$\therefore \dfrac{y-11}{(y-1)(y-7)}$ $y\neq -2,1,7$

PART 2: APPLICATION

16. A rectangular prism has length = $\frac{2x-5}{x+4}$, width = $\frac{3x+2}{3x-1}$, and height = $\frac{x+4}{3x+1}$, all in metres.

a) Determine a simplified expression for the volume of the rectangular prism. Express your answer as a quotient of two polynomials in standard (not factored) form, and state any restrictions.

b) Determine the volume when x = 4 metres.

a) $V=l\times w\times h$

$$V(x)=\frac{(2x-5)}{\cancel{(x+4)}}\cdot\frac{(3x+2)}{(3x-1)}\cdot\frac{\cancel{(x+4)}}{(3x+1)}$$ $x\neq -4,\ x\neq\frac{1}{3},\ x\neq\frac{-1}{3}$

$$V(x)=\frac{(2x-5)(3x+2)}{(3x-1)(3x+1)}$$ $\Rightarrow x\neq\frac{-1}{3},\frac{1}{3},-4$

b) $$V(4)=\frac{(2\cdot 4-5)(3\cdot 4+2)}{(3\cdot 4-1)(3\cdot 4+1)}=\frac{(8-5)(12+2)}{(12-1)(12+1)}=\frac{3\cdot 14}{11\cdot 13}$$

$$V(4)=\frac{42}{143}$$

$\therefore V(4)=0.29\,m^2$

17. There are 2 rational expressions, P/Q and R/S, where Q = $x^2 - 16$, R = x + 2, and
S = $x^2 - x - 12$. If P/Q + R/S = A/B, where A = $6x^2 + 19x + 2$, determine an expression for P.

$$\frac{P}{Q} + \frac{R}{S} = \frac{A}{B}$$

$6x^2 + 19x + 2$ M : 12 A : 19 N : ?

① $\frac{P}{x^2-16} + \frac{x+2}{x^2-x-12} = \frac{6x^2+19x+2}{B}$

② $\frac{P}{(x-4)(x+4)} + \frac{x+2}{(x+3)(x-4)} = \frac{6x^2+19x+2}{B}$ LCD = $(x+3)(x+4)(x-4)$

$5x^2+13x-6$

$\frac{(5x-2)(5x+15)}{5}$

M = −30 A = +13 N = −2, +15

③ $\frac{P(x+3)}{(x-4)(x+4)(x+3)} + \frac{(x+2)(x+4)}{(x+3)(x-4)(x+4)} = \frac{6x^2+19x+2}{B}$

④ $\frac{P(x+3) + x^2+6x+8}{(x-4)(x+4)(x+3)} = \frac{6x^2+19x+2}{B}$

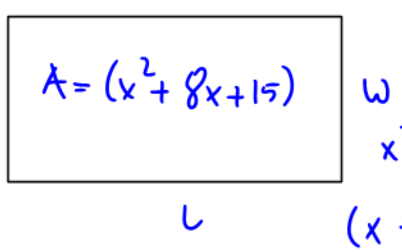
⑤ $\Rightarrow P(x+3) + x^2+6x+8 = 6x^2+19x+2$

$P(x+2) = 5x^2+13x-6$

$P = \frac{5x^2+13x-6}{(x+2)}$

⑥ $P = \frac{(5x-2)(x+3)}{(x+2)}$ $x \neq -2$

18. The area of a rectangular field is given by the expression $x^2 + 8x + 15$.
a) Determine the expressions that represent the dimensions of the field.
b) Determine a fully simplified expression for the perimeter of the field.



a) $A = l \times w$

$x^2+8x+15 = l \times w$

$(x+3)(x+5) = l + w$

∴ Length = $x + 5$

Width = $x + 3$

b) $P = 2(l + w)$

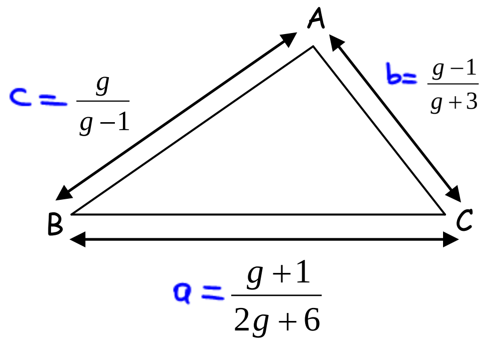
$= 2(x+5+x+3)$

$= 2(2x+8)$

$P = 4x + 16$

PART 3: THINKING

14\. Given triangle ABC below, determine a simplified expression that represents the **perimeter** of ΔABC. State restrictions, if any.



$P = a + b + c$

$P = \frac{g+1}{2g+6} + \frac{g-1}{g+3} + \frac{g}{g-1}$

$= \frac{g+1}{2(g+3)} + \frac{g-1}{g+3} + \frac{g}{g-1}$ $\quad g \neq -3, 1$

$LCD = 2(g+3)(g-1)$

$P = \frac{(g+1)(g-1)}{2(g+3)(g-1)} + \frac{(g-1)(2)(g-1)}{(g+3)(2)(g-1)} + \frac{(g)(2)(g+3)}{(g-1)(2)(g+3)}$

$P = \frac{g^2 - 1 + 2(g^2 - 2g + 1) + 2g^2 + 6g}{2(g-1)(g+3)}$ $\quad g \neq 1, g \neq -3$

$= \frac{2g^2 - 4g + 2 + 3g^2 + 6g - 1}{2(g-1)(g+3)}$

$= \frac{5g^2 + 2g + 1}{2(g-1)(g+3)}$ $\quad g \neq 1, -3$

$5g^2 + 2g + 1$

M : 5
A : 2
N : ?

15. Hanz and Franz are walking 60 km to raise money to fight Breast Cancer. Franz walks 1 km/h faster than Hanz, but stops for 30 min to sign autographs. They start at the same time, but Franz finishes 2 ½ hours before Hanz. How fast was each person walking, and how long did it take for each person to finish the walk?

Let "x" be Hanz's speed

NAME	Distance	Speed	Time
Hanz	60km	x	$\frac{60}{x}$
Franz	60km	$x+1$	$\frac{60}{x+1} + 0.5$

→ stops for autographs

Franz finishes $2\frac{1}{2}$ h (2.5h) before Hanz

$$\underbrace{\frac{60}{x+1} + 0.5}_{\text{Franz}} = \underbrace{\frac{60}{x}}_{\text{Hanz}} - 2.5$$

$$0.5 + 2.5 = \frac{60}{x} - \frac{60}{x+1} \qquad LCD = (x)(x+1)$$

$$3 = \frac{60(x+1)}{x(x+1)} - \frac{60(x)}{(x+1)(x)}$$

$$3 = \frac{\cancel{60x} + 60 - \cancel{60x}}{x^2 + x} \qquad \text{cross multiply}$$

$$3x^2 + 3x = 60$$

$$3x^2 + 3x - 60 = 0$$

$$\frac{\cancel{3}(x^2 + x - 20)}{\cancel{3}} = \frac{0}{3}$$

$$(x-4)(x+5) = 0$$

$$x - 4 = 0 \qquad x + 5 = 0$$

$$x = 4 \qquad x = -5 \rightarrow \text{speed can't be negative}$$

∴ Hanz's speed 4 km/h

Franz's speed 5 km/h