Knowledge	Application	TIPS
/12	/21	/10

Knowledge:

1. Given the points $A(-1,5)$, $B(2,9)$, $C(-4,8)$, then determine the following:

a) the slope of the line passing through AB.

[2] $m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{9-5}{2-(-1)} = \frac{4}{3}$

Answer: $4/3$

b) the slope of the line perpendicular to the line segment AC.

[2] $m_{AC} = \frac{y_2-y_1}{x_2-x_1} = \frac{8-5}{-4-(-1)} = \frac{3}{-4+1} = \frac{3}{-3} = -1$ opp rec → 1

Answer: 1

c) the midpoint of the line segment BC.

$M_{BC}(x,y) = \left(\text{average of } x, \text{average of } y\right)$

[2] $= \left(\frac{-4+2}{2}, \frac{9+8}{2}\right) = (-1, 8.5)$

Answer: $(-1, 8.5)$

d) the exact length of the line segment AC.

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

[2] $= \sqrt{[-1-(-4)]^2+(5-8)^2} = \sqrt{(-1+4)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9}\cdot\sqrt{2}$
$= 3\sqrt{2}$

Answer: $3\sqrt{2}$

2. Determine the equation of the line in **standard form** that is **perpendicular** to the line $3x-6y+8=0$ and passes through the point $P(-1,2)$.

[4] Step 1: Rearrange

$3x-6y+8=0$

$\frac{3x}{6}+\frac{8}{6}=\frac{6y}{6}$

$\boxed{y=\frac{1}{2}x+\frac{4}{3}}$

Step 2: $m=-2$ $P(-1,2)$ (p, q)

$y=m(x-p)+q$

$y=-2[x-(-1)]+2$

$y=-2(x+1)+2$

$y=-2x-2+2$

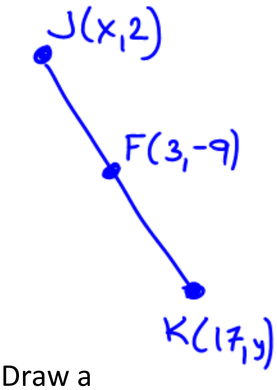
$y=-2x$ rearrange

$\boxed{2x+y=0}$

∴ The equation is $2x+y=0$

Application:

3. The point $F(3,-9)$ is the midpoint of the line segment JK. If endpoint J is located at $(x, 2)$ and K is located at $(17, y)$, find the value of the missing coordinates.



[4]

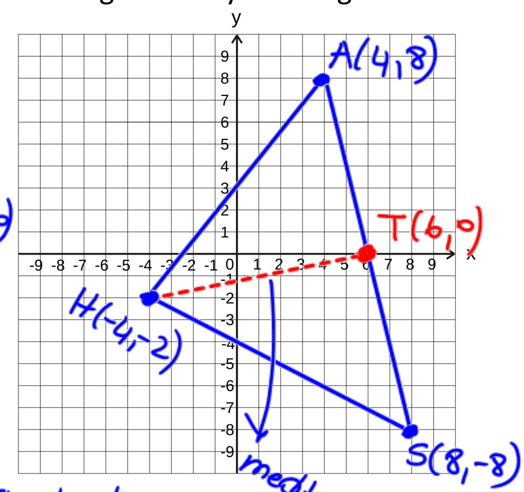
Midpoint $(3,-9)$

x	y
$2 \cdot 3 = \frac{x+17}{2} \cdot 2$	$2 \cdot -9 = \frac{2+y}{2} \cdot 2$
$6 = x+17$	$-18 = 2+y$
$6-17 = x$	$-18-2 = y$
$\boxed{x=-11}$	$\boxed{y=-20}$

$\therefore$ $x = -11$
$y = -20$

4. On the grid below, draw triangle AHS with vertices $A(4,8)$, $H(-4,-2)$, *and* $S(8,-8)$. Draw a labelled diagram of the median from H *to* AS. Determine algebraically the length of the median from H *to* AS.

[4]



Step 1: Midpoint of $\overline{AS}$

$T(x,y) = \left(\frac{4+8}{2}, \frac{8+(-8)}{2}\right) = (6,0)$

Step 2. Length of median $H(-4,-2)$ $T(6,0)$

$d = \sqrt{(-4-6)^2 + (-2-0)^2}$

$= \sqrt{100+4}$

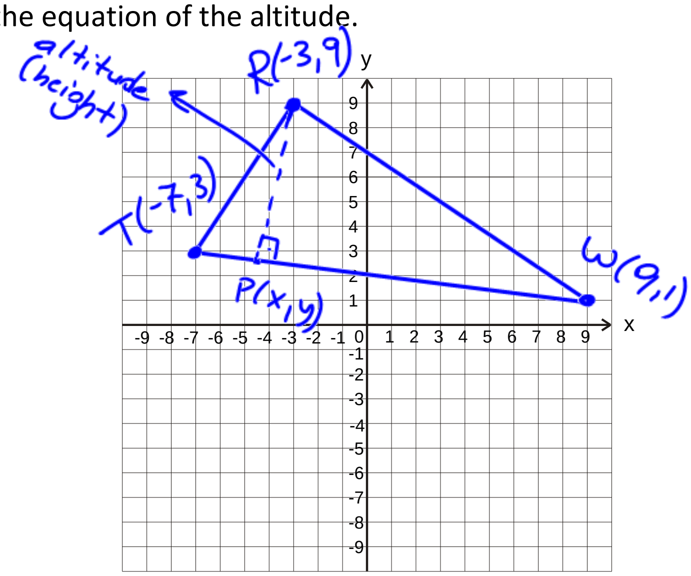
$= \sqrt{104} = \sqrt{4 \cdot 26} = \sqrt{4} \cdot \sqrt{26}$

$= 2\sqrt{26}$

$\therefore$ The length of the median is $2\sqrt{26}$

5. On the grid below, draw triangle TRW with vertices $T(-7,3)$ $R(-3,9)$ $W(9,1)$. Draw a labelled diagram of the altitude from R *to* TW. Determine algebraically the equation of the altitude.

You need ☐ slope ☑ point $R(-3,9)$



[5] 1) Slope of $\overline{RP}$ is opp. rec. of $\overline{TW}$

$m_{TW} = \frac{3-1}{-7-9} = \frac{2}{-16} = \frac{-1}{8}$ $\boxed{m_{RP} = 8}$

2) $y = m(x-p)+q$ $R(-3,9)$ $m_{RP} = 8$

$y = 8[x-(-3)]+9$

$y = 8(x+3)+9$

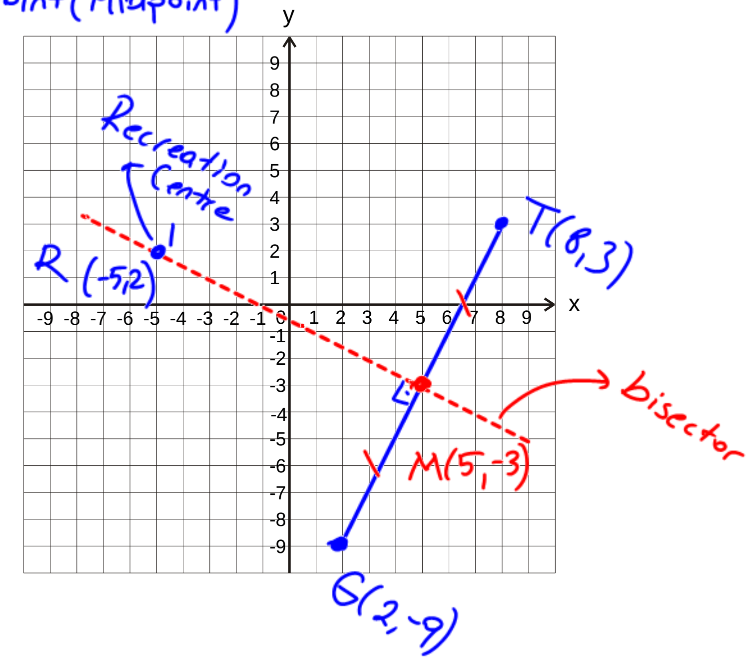
$y = 8x+24+9$

$\boxed{y = 8x+33}$

$\therefore$ The equation of $\overline{RP}$ is $y = 8x+33$

6. The coordinates of two towns are $T(8,3)$ and $G(2,-9)$. Plot and label the two towns on the grid below. Draw a labelled diagram of the perpendicular bisector of the line segment joining these two towns. Determine algebraically the equation of the perpendicular bisector. If the two towns have decided to build a recreation centre at $(-5,2)$, determine if this is a good place to build. Justify your answer. You need ☐ slope ☑ point (Midpoint)

[8]



Step 1: midpoint of $\overline{GT}$

$M(x,y) = \left(\frac{8+2}{2}, \frac{-9+3}{2}\right) = (5,-3)$ ✓

Step 2: The slope of right bisector is opp. reciprocal of $\overline{GT}$

$m_{GT} = \frac{-9-3}{2-8} = \frac{-12}{-6} = 2$

$m_{bisector} = -1/2$

Step 3: $m = -1/2$ $M(5,-3)$

$y = m(x-p)+q$

$y = \frac{-1}{2}(x-5)+(-3)$

$y = \frac{-1}{2}x + \underbrace{\frac{5}{2}}_{2.5} - 3$

$\boxed{y = -0.5x - 0.5}$

We need to see if the recreation centre's distance is equal from both towns.

distance from town $G = \sqrt{(-5-2)^2 + [2-(-9)]^2} = \sqrt{49+121} = \sqrt{170}$

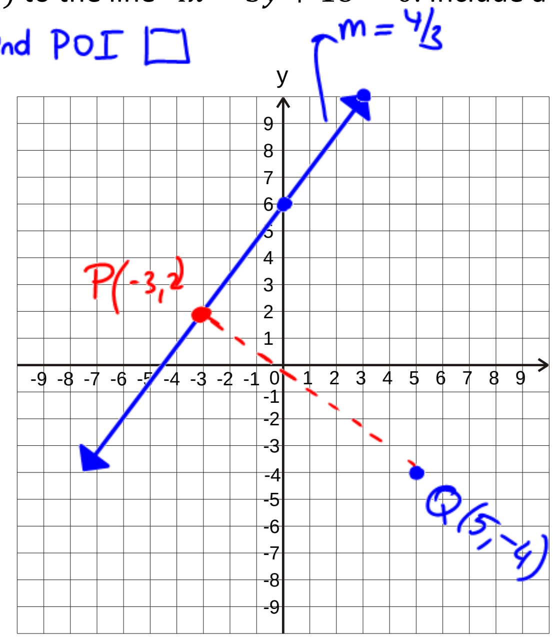
distance from town $T = \sqrt{(-5-8)^2 + (2-3)^2} = \sqrt{169+1} = \sqrt{170}$

if the recreation centre is built on $(-5,2)$, it'll be equidistant from both towns; therefore, it's a good place.

TIPS:

7. Determine the shortest distance from the point $Q(5, -4)$ to the line $4x - 3y + 18 = 0$. Include a fully labelled diagram. Include an algebraic solution. Find POI $\square$

[10]



Step1: Rearrange $4x-3y+18=0$

$$\frac{4x+18}{3} = \frac{3y}{3}$$

$$① \ \boxed{y = \frac{4}{3}x + 6}$$

Step2: Slope of shortest dist.

$m = -3/4 \qquad Q(5,-4)$

$$y = m(x-p)+q$$

$$y = \frac{-3}{4}(x-5) + (-4)$$

$$y = \frac{-3x}{4} + \frac{15}{4} - 4 \Rightarrow y = \frac{-3x}{4} - \frac{1}{4}$$

Step3: Find POI, sub ① → ②

$$\frac{4}{3}x + 6 = \frac{-3x}{4} - \frac{1}{4}$$

collect variables on LS, constants on RS

$$12 \cdot \left(\frac{4x}{3} + \frac{3x}{4} = -6 - \frac{1}{4}\right)$$

multiply the whole equation by 12.
Since you multiply both sides, the equation is still balanced.

$$\frac{48x}{3} + \frac{36x}{4} = -72 - \frac{12}{4}$$

$$16x + 9x = -72 - 3$$

$$\frac{25x}{25} = \frac{-75}{25}$$

$$\boxed{x = -3}$$

sub -3 for x

$$① \ y = \frac{4}{3}x + 6$$

$$y = \frac{4(-3)}{3} + 6$$

$$\boxed{y = 2}$$

Step4: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \qquad P(-3,2) \quad Q(5,-4)$

$$d = \sqrt{(-3-5)^2 + [2-(-4)]^2} = \sqrt{64 + (2+4)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

∴ The shortest distance is 10.