

Day 5- Composite Functions

Warm up: Given $f(x) = 2x + 5$ and $g(x) = \sqrt{x + 1}$

a) Determine $g \circ f$

b) State the domain and range of $g \circ f$

$$= g(f(x))$$

$$= g(2x+5)$$

$$= \sqrt{2x+5+1}$$

$$= \sqrt{2x+6}$$

$$= \sqrt{2(x+3)}$$



$$D: \{x \in \mathbb{R} \mid x \geq -3\}$$

$$R: \{y \in \mathbb{R} \mid y \geq 0\}$$

or $2x+6 \geq 0$

$$2x \geq -6$$

$$x \geq -3$$

EX 1 - Given $f(x) = x^2$ and $g(x) = \sqrt{x}$, state domain and range of:

a) $f \circ g$

b) $g \circ f$

$$= f(g(x))$$

$$= g(f(x))$$

$$= f(\sqrt{x})$$

$$= g(x^2)$$

$$= (\sqrt{x})^2$$

$$= \sqrt{x^2}, \quad x \in \mathbb{R}$$

$$= x, \quad x \geq 0$$

$$= |x| \quad y \geq 0$$

$$y \geq 0$$

EX 2 - Given $f(x) = \sqrt{x} - 2$ and $g(x) = (x-3)^2$;

- a. Determine the simplified equation for the composite function $y = f(g(x))$.

$$y = f((x-3)^2)$$

$$= \sqrt{(x-3)^2} - 2 \rightarrow \text{The least number that is output}$$
$$= |x-3| - 2 \quad \text{is } -2.$$

- b. Determine the **domain** and **range** of the composite function.

$$y = |x-3| - 2$$

$$D: \{x \in \mathbb{R}\} \quad R: \{y \in \mathbb{R} \mid y \geq -2\}$$

EX 3 - Given $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 4$;

- a. Determine the simplified equation for the composite function $y = g(f(x))$.
b. Determine the **domain** and **range** of the composite function.

$$a) \quad y = g(\sqrt{x-1}) = (\sqrt{x-1})^2 + 4$$

$$b) \quad \text{Domain: } \{x \in \mathbb{R} \mid x \geq 1\}$$

$$\text{Range: } \{y \in \mathbb{R} \mid y \geq 4\} \rightarrow \begin{array}{l} x=1 \quad y=4 \\ x>1 \quad y>4 \end{array}$$

EX 4 - Given $h(x) = |x^3 - 1|$, find two functions, f and g , such that $h = f \circ g$

$$f(x) = |x - 1|$$

$$g(x) = x^3$$

or

$$f(x) = |x|$$

$$g(x) = x^3 - 1$$

EX 5 - A banquet hall charges \$850 to rent a reception room, plus \$46.50 per person attending. Next month the banquet hall will be offering a 20% discount off the total bill. Express this discounted cost as a function of the number of people attending.

$$f(x) = 0.8x$$

$$g(x) = 46.50x + 850.$$

$$C = (f \circ g)(x)$$

$$= 0.8(46.50x + 850), \quad x \in \mathbb{N}.$$

Homework: