Day 4 – Composite Functions

$f(6) =$

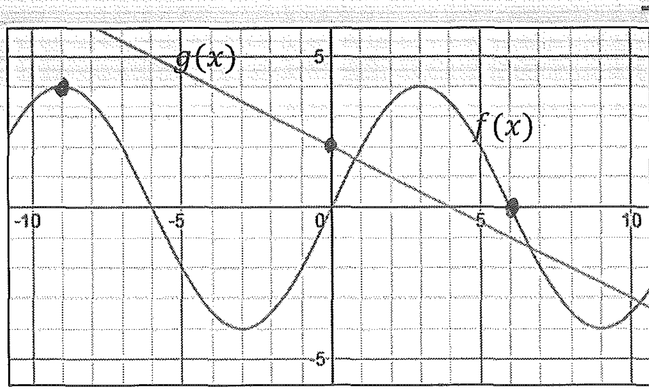
Right to left.

EX 1 - Given the graphs of $f(x)$ and $g(x)$, determine:

a) $f(-9)$

$= 4$

b) $g(f(6)) = g(0) = 2$



A **composite** function is a function made up of (i.e. composed of) other functions
It is formed when one function is ___composed___ into another.

- When $g(x)$ is substituted into $f(x)$, it can be written as: $\boxed{f(g(x))}$ → read as "f of g of x"
- An alternative notation is $f \circ g$ or $(f \circ g)(x) = f(g(x))$
- It can be found by replacing "x" with "$g(x)$" in the expression for "$f(x)$"

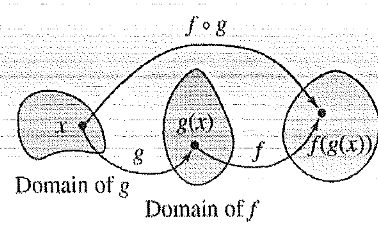
$(f \circ g \circ h)(x) = f[g(h(x))]$

EX 2 - Given $u(x) = x^2 + 3$, $w(x) = x + 4$ and $v(x) = \frac{1}{2x^3}$, determine each composite function

a) $u(w(x))$	b) $w(u(x))$	c) $v(v(x))$	d) $w(u(1))$
$(u \circ w)(x)$			
$= u(x+4)$	$= w(x^2+3)$	$= v\left(\frac{1}{2x^3}\right)$	$= w(1^2+3)$
$= (x+4)^2+3$	$= x^2+3+4$	$= \frac{1}{2\left(\frac{1}{2x^3}\right)^3}$	$= w(4)$
$= x^2+8x+19$	$= x^2+7$	$= \frac{1}{2\left(\frac{1}{8x^9}\right)}$	$= 4+4$
	$w(A) = A+4$	$= 4x^9$	$= 8$
			OR USE part (b)

To determine the **domain** of a composite function:
- Determine the domain of the composite function, and **check that this domain also satisfies the domain of the inner function**

To determine the **range** of a composite function:
- Sketch a graph of the composite function **over the domain found.**



$x-2 \geq 0$

$x \geq 2$

EX 3 – Given $f(x) = -3x + 7$ and $g(x) = \sqrt{x-2} + 1$, determine $f(g(x))$. State the domain and range.

$f(g(x)) = f(\sqrt{x-2}+1)$, $\underline{x \geq 2}$

$= -3(\sqrt{x-2}+1)+7$

$= -3\sqrt{x-2} - 3 + 7$

$= \underline{-3\sqrt{x-2}+4}$ $\rightarrow$ $x=2$ $\quad y=4$

Range: $\{y \in \mathbb{R} \mid y \leq 4\}$

Domain: $\{x \in \mathbb{R} \mid x \geq 2\}$

EX 4 – Given $f(x) = 2x + 1$ and $g(x) = x^2 - 6x$, determine $g(f(x))$. State the domain and range.

$g(f(x)) = g(2x+1)$

$= (2x+1)^2 - 6(2x+1)$

$= 4x^2 + 4x + 1 - 12x - 6$

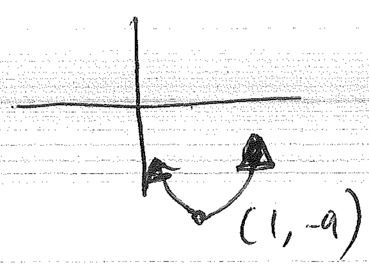
$= (4x^2 - 8x) - 5$ $\qquad$ D: $\{x \in \mathbb{R}\}$

$= 4(x^2 - 2x + 1 - 1) - 5$ $\qquad$ $\left(\frac{\text{middle term}}{2}\right)^2$

$= 4(x^2 - 2x + 1) - 5 - 4$

$= 4(x-1)^2 - 9$

R: $\{y \in \mathbb{R} \mid y \geq -9\}$



$\sin^{-1}$ $\sin x = \sin^{-1} 0.5$

EX 5 - Given $j(x) = x^2 + 8$ and $k(x) = x + 2$, find the equation of the following

a) $k(k^{-1}(x))$ $k^{-1}(x) = x - 2$ b) $j(j^{-1}(x))$

$= x$

$= k(x-2)$

$= x - \cancel{2} + \cancel{2}$

$= x$

$(f \circ f^{-1})(x) = x$

Practice:

1. If $f(x) = \sqrt{x}$ and $g(x) = x + 5$, find each of the following:

a. $f(g(x))$	b. $g(f(x)$	c. $f(g(4))$	d. $g(f(4))$
$= f(x+5)$	$= g(\sqrt{x})$	$= f(4+5)$	$= g(\sqrt{4})$
$= \sqrt{x+5}$	$= \sqrt{x} + 5$	$= f(9)$	$= g(2)$
		$= \sqrt{9}$	$= 7$
		$= 3$	

2. Use the functions $f(x) = 3x + 1$, $g(x) = x^3$, $h(x) = \frac{1}{x+1}$, and $u(x) = \sqrt{x}$, find expressions for the indicated composite functions:

a. $f \circ u = f(u) = f(\sqrt{x})$

$= 3\sqrt{x} + 1$

b. $h \circ g = h(g(x)) = h(x^3)$

$= \frac{1}{x^3 + 1}$

c. $(f \circ g) \circ u = f(g(u)) = f(g(\sqrt{x}))$

$= f((\sqrt{x})^3)$

$= 3(\sqrt{x})^3 + 1$

3. If $f(x) = \sqrt{2-x}$ and $f(g(x)) = \sqrt{2-x^3}$, then what is $g(x)$?

$$g(x) = x^3$$

4. If $g(x) = \sqrt{x} + 7$ and $f(g(x)) = (\sqrt{x}+7)^2$, then what is $f(x)$?

$$f(x) = x^2$$

5. Given $f(x) = -2x + 1$ and $g(x) = \sqrt{x-1} + 1$, determine $f(g(x))$. State the domain and range.

$$f(g(x)) = f(\sqrt{x-1} + 1)$$

$$= -2(\sqrt{x-1} + 1) + 1$$

$$= -2\sqrt{x-1} - 2 + 1$$

$$= -2\sqrt{x-1} - 1$$

D: $\{x \in \mathbb{R} \mid x \geq 1\}$

R: $\{y \in \mathbb{R} \mid y \leq -1\}$

6. Given $h(x) = x^3 - 2$, determine $h(h^{-1}(x)) = x$

$h(x) = x^3 - 2$

$x = y^3 - 2$

$x + 2 = y^3$

$y = \sqrt[3]{x+2}$

$h^{-1}(x) = \sqrt[3]{x+2}$

$h(h^{-1}(x)) = h(\sqrt[3]{x+2})$

$= (\sqrt[3]{x+2})^3 - 2$

$= x + 2 - 2$

$= x$

Homework: