Day 7: 7.1– Solving Exponential Equations

Warm Up: simplify the following using logarithm laws. State any restrictions if necessary.

$\log(x^2 - 3x - 4) - \log(x^2 - 1)$

$= \log\left(\frac{x^2-3x-4}{x^2-1}\right)$

$= \log\left(\frac{(x-4)(x+1)}{(x-1)(x+1)}\right)$

$= \log\frac{(x-4)}{(x-1)}$

$x^2-3x-4 > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$x^2-1>0$

$+ \quad -1 \quad - \quad 1 \quad +$

$\therefore \; x < -1 \;$ or $\; x > 4$

We have seen in lesson 6.1 how changing the base of one or more exponential expressions is a useful technique for solving exponential equations. If there is **no common base**, we have other techniques.

<u>To solve an exponential equation without a common base:</u>

- Take logarithm of both sides and *apply power law*
- Rewrite from exponential to logarithmic form or vice versa
- If a quadratic equation is obtained, apply factoring (or quadratic formula) techniques

*Some solutions lead to **extraneous roots**, which are not valid solutions to the original equation

EX 1 – Solve for x. ***Identify and reject any extraneous roots.***

a) $100 = 50(1.03)^{2x}$

$1.03^{2 \cdot x} = 2$

$\log 1.03^{(2 \cdot x)} = \log 2$

$2x = \frac{\log 2}{\log 1.03}$

$= \frac{\log 2}{2 \log 1.03}$

$= \frac{\log 2}{\log 1.03^2}$

OR.

$1.03^{2x} = 2$

$\log_{1.03} 2 = 2x$

$\boxed{x = \frac{\log_{1.03} 2}{2}}$

b) $4^{2x-1} = 3^{x+2}$

Sol^{n}1 :

$$\log 4^{2x-1} = \log 3^{x+2}$$

$$(2x-1)\log 4 = (x+2)\log 3$$

$$2x\log 4 - \log 4 = x\log 3 + 2\log 3$$

$$2x\log 4 - x\log 3 = 2\log 3 + \log 4$$

$$x\left[\log \frac{4^2}{3}\right] = \log 3^2 \cdot 4$$

$$x = \frac{\log\left[3^2 \cdot 4\right]}{\log(4^2/3)} = \frac{\log(36)}{\log(16/3)}$$

Sol^{n}2

$$4^{2x}4^{-1} = 3^x 3^2$$

$$\frac{(4^2)^x}{3^x} = \frac{3^2}{4^{-1}}$$

$$\Rightarrow \left(\frac{4^2}{3}\right)^x = 3^2 \cdot 4$$

$$x = \frac{\log\left[3^2(4)\right]}{\log\left[4^2/3\right]}$$

$$= \frac{\log 36}{\log 16/3}$$

c) $3^{2x} + 3^x - 12 = 0$

$$p^2 + p - 12 = 0 \quad \left[p = 3^{2x}\right]$$

$$(p+4)(p-1) = 0$$

$p = -4$

$3^{2x} = -4$

No sol^n

$p = 1$

$3^{2x} = 1$

$3^{2x} = 3^0$

$2x = 0$

$x = 0$

d) $5^x - 20(5)^{-x} = 1$ $\Rightarrow$ $5^x - \frac{20}{5^{+x}} = 1$

$P = 5^x$

$P - \frac{20}{P} = 1$

$P^2 - 20 = P$

$P^2 - P - 20 = 0$

$(P+4)(P-5) = 0$

$P = -4 \quad P = 5$

$\therefore 5^x = -4$

No sol^n

$5^x = 5$

$\boxed{x = 1}$

To solve any growth or decay word problems, use the following equation: $A = A_0(1 \pm r)^n$

Where..

A represents ***final amount*** — *A_0 represents* ***initial amount***

(+) represents ***growth*** — *(-) represents* ***decay***

r represents ***growth rate*** — *n represents* ***number of growth/decay periods***

* Half -life: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ — *where h represents* ***half life***

* Doubling time: $A = A_0(2)^{\frac{t}{d}}$ — *where d represents* ***doubling time***

EX 2 - ***Half-life*** is the time taken for a radioactive substance to decay to half of its initial amount. A 50-mg sample of radioactive iodine decays to 23 mg after 12 minutes. Determine the half-life of radioactive iodine.

$A = a\left(\frac{1}{2}\right)^{t/h}$

$23 = 50\left(\frac{1}{2}\right)^{\frac{12}{h}}$

$0.46 = (0.5)^{12/h}$

$\frac{12}{h} = \frac{\log 0.46}{\log 0.5}$

$h = \frac{(12)(\log 0.5)}{\log 0.46} = 10.71$ minutes

$\therefore$ half life was 12.71 minutes.