Day 6 – Solving Inequalities

EX 1 - Let $f(x) = x$ and $g(x) = (x-2)^2$. Solve $f(x) > g(x)$ both graphically and algebraically.

$x > (x-2)^2$

$x > x^2 - 4x + 4$

$x^2 - 5x + 4 < 0$

$(x-4)(x-1) < 0$

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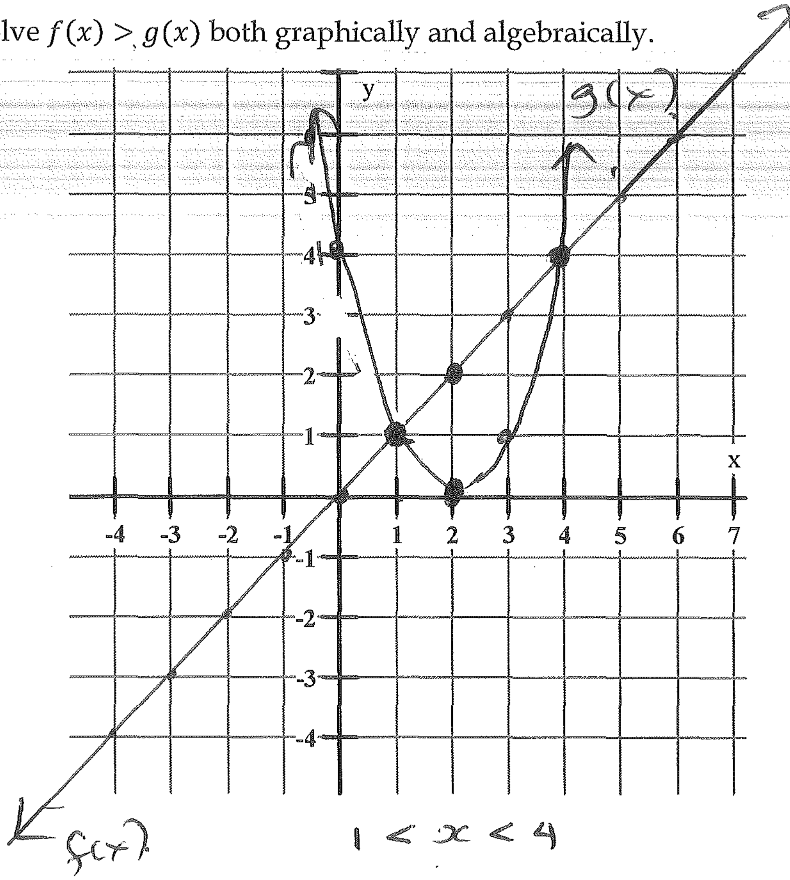
1 4

$x \in (1, 4)$

$1 < x < 4$



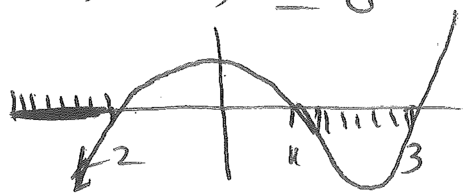
$1 < x < 4$

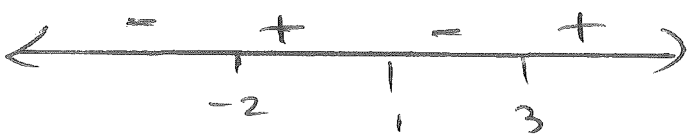
EX 2: Let $f(x) = x^3 - 3x^2 - 4x$ and $g(x) = -x^2 + x - 6$. Solve $f(x) \leq g(x)$ algebraically.

$$x^3 - 3x^2 - 4x \leq -x^2 + x - 6$$

$$x^3 - 2x^2 - 5x + 6 \leq 0$$

$$(x-1)(x-3)(x+2) \leq 0$$





$$x \in (-\infty, -2] \cup [1, 3]$$

Let $P(x) = x^3 - 2x^2 - 5x + 6$

$P(1) = 0 \Rightarrow (x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & \downarrow & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$\therefore P(x) = (x-1)(x^2 - x - 6)$

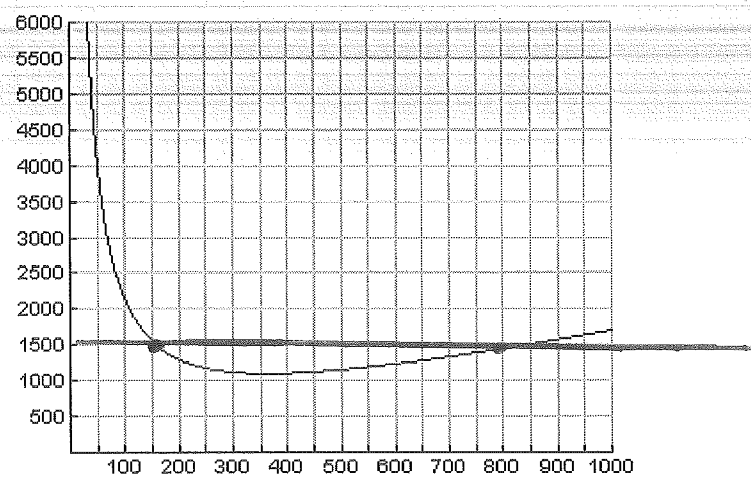
$= (x-1)(x-3)(x+2)$

EX 3: A computer store's cost, C, for shipping and storing n computers can be modeled by the function $C(n) = 1.5n + \dfrac{200000}{n}$. The storage capacity of the store's warehouse is 750 units.

a) The function is graphed to the right. What is the domain of the function in the context of the question?

$\{n \in \mathbb{N},\ 0 < n \leq 750\}$

$n \in (0, 750],\ n \in \mathbb{N}$



b) Determine the number of computers that should be shipped or stored to keep costs below \$1500. Prove algebraically. Graphically: $150 < n \leq 750$

$$1.5n + \frac{200\,000}{n} < 1500$$

$$n\left(1.5n + \frac{200000}{n} - 1500\right) < 0.$$

$$1.5n^2 - 1500n + 200{,}000 < 0$$

USE QF $n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n_1 = 158.4 \qquad n_2 = 841.5$

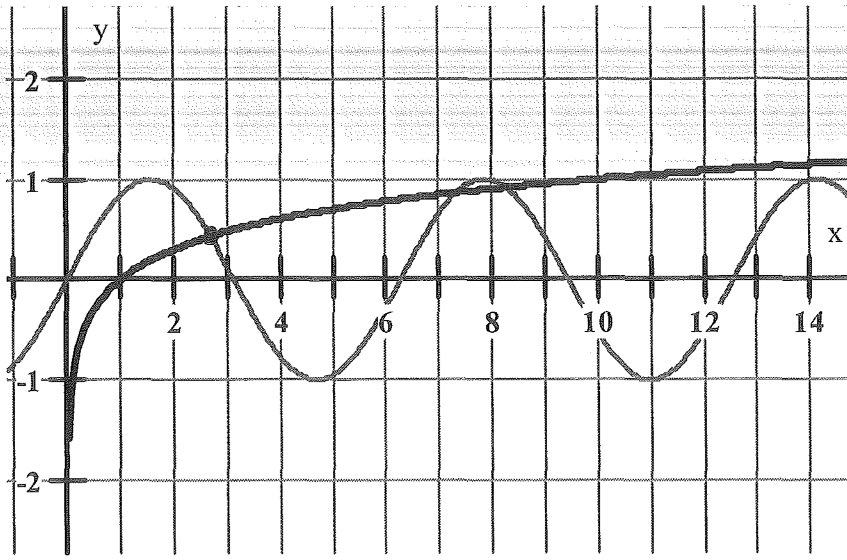
$+ \qquad -$

$0 \qquad 159 \qquad 750$

$$159 < n \leq 750$$

$\sin x > \log x$

EX 4 – Below, $f(x) = \sin x$ and $g(x) = \log x$ are graphed. Determine when $f(x) > g(x)$



$0 \leq x < 2.7$

$7.5 < x < 8.2$

Homework: