Day 4: 2.4 The Quotient Rule

Rule 6 The Quotient Rule

At a point where $v \neq 0$, the quotient $y = \frac{u}{v}$ of the two differentiable functions is differentiable, and $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ OR $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Proof of Rule 6 (see page 85-86 in the textbook)

Let $y = \frac{f(x)}{g(x)} = f(x)[g(x)]^{-1}$

$y' = f'(x)[g(x)]^{-1} + f(x)(-1)[g(x)]^{-2}g'(x)$ ← factor $[g(x)]^{-2}$

$= g(x)^{-2}[f'(x)g(x) - f(x)g'(x)]$

$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Example of Rule 6

a. What is the derivative of $f(x) = \frac{x^2 - 1}{x^2 + 1}$?

$f'(x) = \frac{2x(x^2+1) - (x^2-1)(2x)}{(x^2+1)^2}$ ↗ expand and simplify the numerator

$= \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2}$

$= \frac{4x}{(x^2+1)^2}$

b. $y=\dfrac{3x-2}{5x+1}$

$$y' = \frac{3(5x+1) - (3x-2)(5)}{(5x+1)^2}$$

$$= \frac{15x+3-15x+10}{(5x+1)^2}$$

$$= \frac{13}{(5x+1)^2}$$

c. $f(t)=\dfrac{t^2+2t+5}{t^2-5t+1}$

$$f'(t) = \frac{(2t+2)(t^2-5t+1) - (2t-5)(t^2+2t+5)}{(t^2-5t+1)^2}$$

$$= \frac{\cancel{2t^3}-10t^2+2t+2t^2-10t+2-\cancel{2t^3}-4t^2-10t+5t^2+10t+25}{(t^2-5t+1)^2}$$

$$= \frac{-7t^2-8t+27}{(t^2-5t+1)^2}$$

d. $y=\dfrac{x^2+3x-2}{\sqrt{x}}$

$$= x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$$

$$= \frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$$

$$= \frac{3\sqrt{x}}{2} + \frac{3}{2\sqrt{x}} + \frac{1}{\sqrt{x^3}}$$

e.

$y=\dfrac{4x^3-8x}{2x}$

$y = 2x^2 - 4$

$y' = 4x$

f.

$y=\dfrac{9}{x^2+3x-1}$

$y = 9(x^2+3x-1)^{-1}$

$y' = 9(x^2+3x-1)^{-2}(2x+3)$

$= \dfrac{-9(2x+3)}{(x^2+3x-1)^2}$

g.

$y = \dfrac{5x^4}{7}$

$y' = \frac{5}{7}(4x^3)$

$= \dfrac{20x^3}{7}$

Rule 7 Power Rule for Negative Integer Powers of x.

If n is a negative integer and $x \neq 0$, then $\frac{d}{dx}(x^n) = nx^{n-1}$

Example of Rule 7

What is the derivative of $f(x) = x^{-5}$?

Using Rule 7

$y' = -5x^{-6}$

$= \frac{-5}{x^6}$

Using Quotient Rule

$f(x) = \frac{1}{x^5}$

$f'(x) = \frac{(1)'(x^5) - (1)(x^5)'}{(x^5)^2}$

$= \frac{0 - 1(5x^4)}{x^{10}}$

$= \frac{-5}{x^6}$

$y = 4x^{-4}$

$y' = -16x^{-5}$

$= \frac{-16}{x^5}$

$f(x) = \sqrt[3]{\frac{1}{x^2}}$

$f(x) = \left(\frac{1}{x^2}\right)^{\frac{1}{3}} = x^{-\frac{2}{3}}$

$f'(x) = -\frac{2}{3} x^{-\frac{2}{3}-1} = -\frac{2}{3} x^{-\frac{5}{3}}$

$= \frac{-2}{3\sqrt[3]{x^5}}$

What do you notice about this rule?