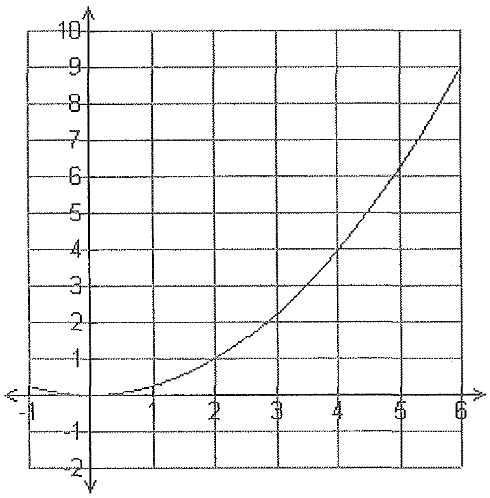
Day 6: 1.2. The Slope of the tangent

<u>INVESTIGATION:</u>

Find the slope of the tangent line of $f(x) = \frac{1}{4}x^2$ at P(2, 1)



Start with Q(6, 9) $IROC = \frac{9-1}{6-2} = \frac{8}{4} = 2$

(2,1) (6,9)

Move Q closer to (4, 4) $IROC = \frac{4-1}{4-2} = \frac{3}{2}$

(2,1) (4,4)

Even closer Q (2.1, 1.1025) $IROC = \frac{1.1025-1}{2.1-2}$

(2,1) (2.1, 1.1025)

$= 1.025$

Try Q(2 + h, ¼(2 + h)²)

$$IROC = \frac{\frac{1}{4}(2+h)^2 - 1}{2+h-2} = \frac{\frac{1}{4}(4+4h+h^2)-1}{h}$$

$$= \frac{\frac{1}{4}h^2 + h + \cancel{1} - \cancel{1}}{h} = \frac{\cancel{h}(\frac{1}{4}h+1)}{\cancel{h}}$$

$h \to 0$ $IROC = 1 \to$ exact answer.

Slope of a Tangent as a Limit

The slope of the tangent to the graph $y = f(x)$ at point $P(a, f(a))$ is
$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, **if this limit exists.**

Tangent to a Curve – a bit of History

"The problem of how to find a tangent to a curve became the dominant mathematical problem of the early 17th century and it is hard to overestimate how badly scientists of the day wanted to know the answer. Descartes went so far as to say that the problem was the most useful and most general problem not only that he knew but that he had any desire to know."

<u>This limit is one of the two most important mathematical objects considered in Calculus</u>

Example. 1: Determine the slope and the equation of the tangent line to the curve $y = x^2+4x+1$ at $x = 3$

$$m = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0} \frac{h^2+10h+\cancel{22}-\cancel{22}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+10)}{\cancel{h}} = 10$$

$$f(3+h) = (3+h)^2+4(3+h)+1$$

$$= 9+6h+h^2+12+4h+1$$

$$= h^2+10h+22$$

$$f(3) = 3^2+4(3)+1$$

$$= 22$$

eq^n: $y - y_1 = m(x-x_1) \Rightarrow y-22 = 10(x-3)$

$$y = 10x - 8$$

Example 2: Determine the slope and the equation of the tangent line to the rational function $f(x) = \frac{3x+6}{x}$ at the point (2, 6) $m = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$$m = \lim_{h\to 0} \left(\frac{3(2+h)+6}{2+h} - \frac{6}{1}\right) \div h$$

$$= \lim_{h\to 0} \left(\frac{6+3h+6}{2+h} - \frac{6}{1}\right) \times \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{3h+12-6(2+h)}{(2+h)(1)(h)} = \lim_{h\to 0} \frac{3h+\cancel{12}-\cancel{12}-6h}{(2+h)(1)(h)}$$

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{(2+h)(1)(\cancel{h})} = -\frac{3}{2}$$

$$y - y_1 = m(x-x_1)$$

$$y - 6 = -\frac{3}{2}(x-2)$$

$$y - 6 = -\frac{3}{2}x + 3$$

$$\boxed{y = -\frac{3}{2}x + 9}$$

eqn of tangent.

Example 3: Determine the slope and the equation of the tangent line to the function $f(x) = \sqrt{x}$ at $x = 9$

$$m = \lim_{h\to 0} \frac{f(9+h)-f(9)}{h}$$

multiply by the conjugate

$$= \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}$$

$$= \lim_{h\to 0} \frac{\cancel{9}+\cancel{h}-\cancel{9}}{\cancel{h}(\sqrt{9+h}+3)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3}$$

$$= \frac{1}{6}$$

$$y - y_1 = m(x-x_1)$$

$$y - 3 = \frac{1}{6}(x-9)$$

$$y = \frac{1}{6}x - \frac{9}{6} + 3$$

$$y = \frac{1}{6}x - \frac{9}{6} + \frac{18}{6}$$

$$= \frac{1}{6}x + \frac{9}{6}$$

$$= \frac{1}{6}x + \frac{3}{2}$$