Day 4: Evaluating Limits

Indeterminate Form $\frac{0}{0}$

Using factoring:

a) $\lim\limits_{t\to 2}\frac{t^2-3t+2}{t-2}$

$= \lim\limits_{t\to 2}\frac{\cancel{(t-2)}(t-1)}{\cancel{t-2}}$

$= 2-1 = 1$

b) $\lim\limits_{x\to b}\frac{x^5-b^5}{x^{10}-b^{10}} = \lim\limits_{x\to b}\frac{\cancel{(x^5-b^5)}}{(x^5+b^5)\cancel{(x^5-b^5)}}$

$= \frac{1}{b^5+b^5} = \frac{1}{2b^5}$

Using rationalizing (conjugate):

a) $\lim\limits_{t\to 0}\frac{\sqrt{t+2}-\sqrt{2}}{t} \cdot \frac{\sqrt{t+2}+\sqrt{2}}{\sqrt{t+2}+\sqrt{2}}$

$= \lim\limits_{t\to 0}\frac{\cancel{t}+\cancel{2}-\cancel{2}}{\cancel{t}(\sqrt{t+2}+\sqrt{2})}$

$= \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$

b) $\lim\limits_{x\to 0}\frac{2-\sqrt{4+x}}{x} \cdot \frac{2+\sqrt{4+x}}{2+\sqrt{4+x}}$

$= \lim\limits_{x\to 0}\frac{4-(4+x)}{(x)(2+\sqrt{4+x})}$

$= \lim\limits_{x\to 0}\frac{-\cancel{x}}{\cancel{x}(2+\sqrt{4+x})} = -\frac{1}{4}$

Using substitution:

a) $\lim\limits_{x\to 0}\frac{(x+8)^{\frac{1}{3}}-2}{x}$

Let $u = (x+8)^{\frac{1}{3}}$

$u^3 = x+8$

$u^3 - 8 = x$

$x\to 0 \quad u\to 2$

$\lim\limits_{u\to 2}\frac{u-2}{u^3-8} = \lim\limits_{u\to 2}\frac{\cancel{(u-2)}}{\cancel{(u-2)}(u^2+2u+4)}$

$= \frac{1}{12}$

b) $\lim\limits_{x\to 27}\frac{27-x}{x^{\frac{1}{3}}-3}$

Let $p = x^{\frac{1}{3}}$

$p^3 = x$

$x\to 27 \quad p\to 3$

$= \lim\limits_{p\to 3}\frac{27-p^3}{p-3}$

$= \lim\limits_{p\to 3} -\frac{(p^3-27)}{p-3}$

$= \lim\limits_{p\to 3} -\frac{\cancel{(p-3)}(p^2+3p+9)}{\cancel{(p-3)}}$

$= -27$

Note: Definition of absolute value $|x| = \begin{cases} x, x \geq 0 \\ -x, \ x < 0 \end{cases}$

Absolute value:

$|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases}$

a) $\lim\limits_{x\to 2} \frac{|x-2|}{x-2}$

$\lim\limits_{x\to 2^+} \frac{|x-2|}{x-2} = \lim\limits_{x\to 2^+} \frac{(x-2)}{(x-2)} = 1$

$\lim\limits_{x\to 2^-} \frac{|x-2|}{x-2} = \lim\limits_{x\to 2^-} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}} = -1$

$\therefore \lim\limits_{x\to 2} \frac{|x-2|}{x-2}$ d.n.e

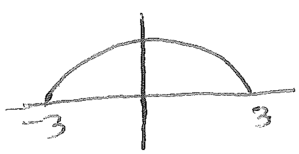
b) $\lim\limits_{x\to 5} \frac{|x-5|}{x^2-25}$

$= \lim\limits_{x\to 5^-} \frac{-\cancel{(x-5)}}{\cancel{(x-5)}(x+5)}$

$= \lim\limits_{x\to 5^-} \frac{-1}{x+5}$

$= -\frac{1}{10}$

Special cases:



a) $\lim\limits_{x\to 3^-} \sqrt{9-x^2}$

$= 0$

b) $\lim\limits_{x\to 3} \sqrt{9-x^2}$ is not defined

since $\lim\limits_{x\to 3^+} \sqrt{9-x^2}$ does not exist

c) $f(x) = \begin{cases} x+4, & \text{if } x < 1 \\ -x, & \text{if } x \geq 1 \end{cases}$; $\lim\limits_{x\to 1} f(x)$

check if there is a jump at $x=1$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (x+4) = 5$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (-x) = -1$

implies $\lim\limits_{x\to 1} f(x)$ d.n.e