Day 1/2: Do you remember…?

Equation of a line

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation

$$y = mx + b$$

Formula

$$y - y_1 = m(x - x_1)$$

We need at least 2 points or slope and 1 point. (to define a linear line)

Example 1 Determine the equation of the line:

a) With slope 3 and y-intercept -2 $\quad y = 3x - 2$

b) Through points $(-3, 1)$ and $(-5, 7)$ $\quad m = \frac{7-1}{-5+3} = \frac{6}{-2} = -3$

$$y - y_1 = m(x - x_1) \Rightarrow y - 7 = -3(x+5)$$
$$y = -3x - 8$$

Function Notation

$$f(x) = \frac{2x-3}{x^2 - 2x + 1}$$

f(-2) = $\frac{2(-2)-3}{(-2)^2 - 2(-2)+1}$ $= \frac{-7}{9}$

f(0) = $\frac{2(0)-3}{0^2 - 2(0)+1}$ $= -3$

f(2x) = $\frac{2(2x)-3}{(2x)^2 - 2(2x)+1}$ $= \frac{4x-3}{4x^2-4x+1}$

f(2x − 1) = $\frac{2(2x-1)-3}{(2x-1)^2 - 2(2x-1)+1}$ $= \frac{4x-5}{4x^2-8x+4}$

$$f(x) = \begin{cases} \sqrt{3-x}, \text{ if } x < 0 \\ \sqrt{3+x}, \text{ if } x \geq 0 \end{cases}$$

f(-2) = $\sqrt{3-(-2)}$ $= \sqrt{5}$

f(0) = $\sqrt{3+0}$ $= \sqrt{3}$

f(2x) = $\begin{cases} \sqrt{3-2x}, & x<0 \\ \sqrt{3+2x}, & x \geq 0 \end{cases}$

f(x²) = $\sqrt{3+x^2}$ since $x^2 \geq 0$ for $x \in \mathbb{R}$

Exponent laws

- Basic laws

$$a^m \cdot a^n = a^{m+n} \qquad (a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

- Negative exponent

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$$

Expanding and Factoring

- Expanding and collect like terms
  eg. $(2x + 4y)(x - y) =$ $2x^2 - 2xy + 4xy - 4y^2 = 2x^2 + 2xy - 4y^2$

- Common factoring – take out the GCF
  e.g. $4x + 12x^2 + 16x^3 =$ $4x(1 + 3x + 4x^2)$

- Difference of Squares: $a^2 - b^2 = (a - b)(a + b)$
  e.g. $36x^4 - 25a^2b^4 =$ $(6x^2 - 5ab^2)(6x^2 + 5ab^2)$

- Simple Trinomials - $x^2 + bx + c$
  e.g. $x^2 - 5x + 6 =$ $(x-2)(x-3)$

- Complex Trinomials - $ax^2 + bx + c$
  e.g. $6x^2 - 5x - 4 =$ $6x^2 - 8x + 3x - 4$

M	A	N
-24	-5	-8, 3

$$= 2x(3x-4) + (3x-4)$$
$$= (2x+1)(3x-4)$$

- Sum and Difference of 2-Cubes:
  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  e.g. $8x^3 - 125 =$ $(2x - 5)(4x^2 + 10x + 25)$

- Factor Theorem:
  $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

eg $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$

Rationalizing Denominators/Graphing

Example 1: Remember cos~~45°~~ or cos$\frac{\pi}{4}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sqrt{2}$ 1 45 1

The **conjugate:** (a + b) and (a – b) (normally binomials with different (middle) sign)

What is (a + b)(a – b) = $a^2 - b^2$

$(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5}) =$ $3 - 5 = -2$

Example 2: We use conjugate to rationalize denominators:

a) $\frac{3}{\sqrt{6}+\sqrt{2}} = \frac{3}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$= \frac{3\sqrt{6}-3\sqrt{2}}{6-2} = \frac{3\sqrt{6}-3\sqrt{2}}{4}$

$\frac{3}{5-\sqrt{2}} = \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{3(5+\sqrt{2})}{25-2}$

$= \frac{3(5+\sqrt{2})}{23}$

b) $\frac{3}{2\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}-\sqrt{3}}$

$= \frac{6\sqrt{5}-3\sqrt{3}}{20-3} = \frac{6\sqrt{5}-3\sqrt{3}}{17}$

$\frac{4\sqrt{3}-\sqrt{2}}{\sqrt{5}-2\sqrt{3}} = \frac{\sqrt{5}+2\sqrt{3}}{\sqrt{5}+2\sqrt{3}}$

$= \frac{4\sqrt{15}+8(3)-2\sqrt{5}-2\sqrt{6}}{5-12} = \frac{24+4\sqrt{15}-2\sqrt{5}-2\sqrt{6}}{-7}$

Example 3: A function f is defined by $f(x) = \begin{cases} (x-1)^2 - 2, & x \geq 1 \\ 2x - 1, & x < 1 \end{cases}$

Sketch the function and then find each.

a) $f(3) = (3-1)^2 - 2$

$= 2^2 - 2$

$= 2$

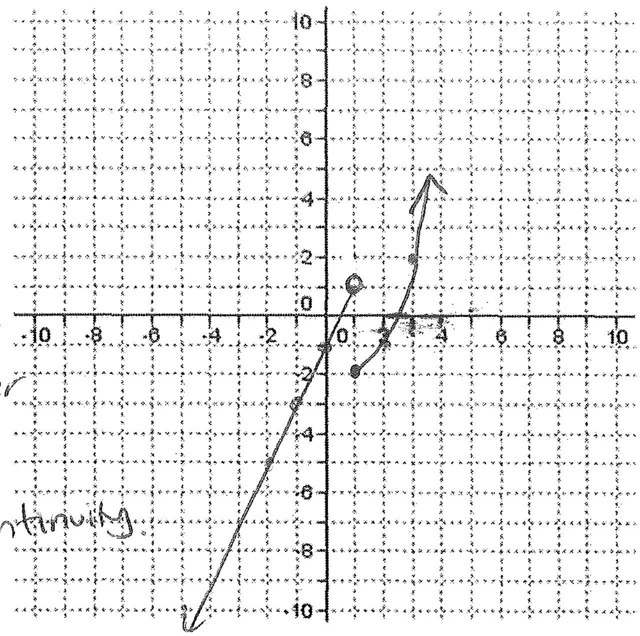
b) $f(1) = (1-1)^2 - 2 = -2$

c) $f(-2) = 2(-2) - 1$

$= -4 - 1$

$= -5$

Please note:
y-values differ
when x=1
It is called
jump discontinuity.



Practice (Rationalize the Numerator/Denominator) (Source Page 9 of Calculus and Vectors, Nelson):

1. *Rationalize the denominator of each expression. Write your answer in the simplest form.*

a. $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6}+\sqrt{10}}{2}$

b. $\frac{3\sqrt{5}-\sqrt{2}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{3\sqrt{10}-2}{4}$

c. $\frac{3}{\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \frac{3\sqrt{5}+3\sqrt{2}}{5-2}$

$= \frac{3\sqrt{5}+3\sqrt{2}}{3} = \sqrt{5}+\sqrt{2}$

d. $\frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$

e. $\frac{2\sqrt{6}}{3\sqrt{27}-\sqrt{8}} = \frac{2\sqrt{6}}{9\sqrt{3}-2\sqrt{2}} \cdot \frac{9\sqrt{3}+2\sqrt{2}}{9\sqrt{3}+2\sqrt{2}}$

$= \frac{18\sqrt{18}+4\sqrt{12}}{81(3)-4(2)} = \frac{18\sqrt{9}\sqrt{2}+4\sqrt{4}\sqrt{3}}{243-8}$

$= \frac{54\sqrt{2}+8\sqrt{3}}{235}$

2. *Rationalize the numerator of each of the following.*

a. $\frac{\sqrt{a}-2}{a-4} \cdot \frac{\sqrt{a}+2}{\sqrt{a}+2}$

$= \frac{\cancel{a-4}}{\cancel{(a-4)}(\sqrt{a}+2)}$

$= \frac{1}{\sqrt{a}+2}$

b. $\frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$

$= \frac{\cancel{x}+\cancel{4}-\cancel{4}}{\cancel{x}(\sqrt{x+4}+2)}$

$= \frac{1}{\sqrt{x+4}+2}$

c. $\frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$

$= \frac{1}{\sqrt{x+h}+\sqrt{x}}$

Practice: Piecewise Functions

Part I. Carefully graph each of the following.

1. $$f(x)=\begin{cases} x+5 & x<-2 \\ -2x-1 & x\geq -2 \end{cases}$$

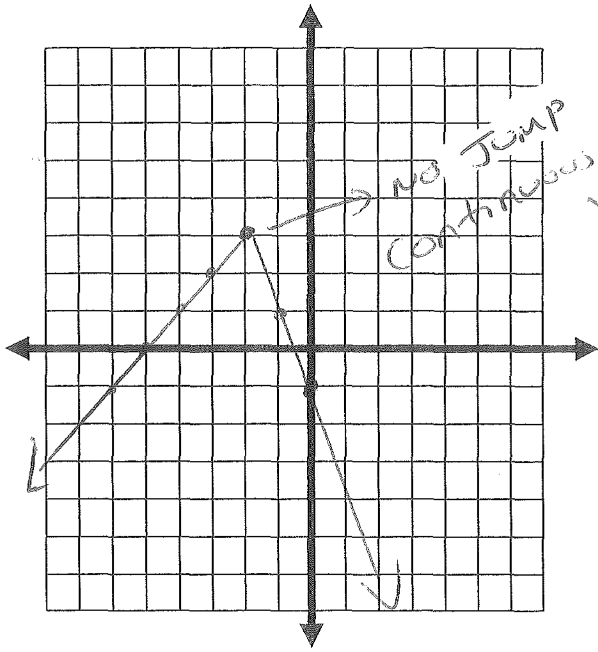
Function? Yes or No

$f(3) =$ $-2(3)-1=-7$

$f(-4) =$ $-4+5=1$

$f(-2) =$ $-2(-2)-1$

$=3$



No Jump
Continuous at $x=-2$

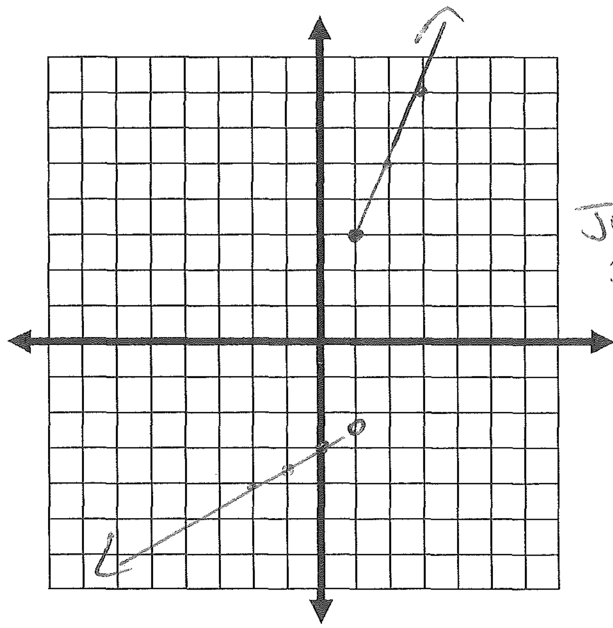
2. $$f(x)=\begin{cases} 2x+1 & x\geq 1 \\ \dfrac{x}{2}-3 & x<1 \end{cases}$$

Function? Yes or No

$f(-2) =$ -4

$f(6) =$

$f(1) =$



Jump
Discontinuity
at $x=1$

3. $$f(x)=\begin{cases} -x+4 & x\leq 0 \\ \dfrac{2x}{3}-1 & 0<x\leq 5 \\ 2 & x>5 \end{cases}$$

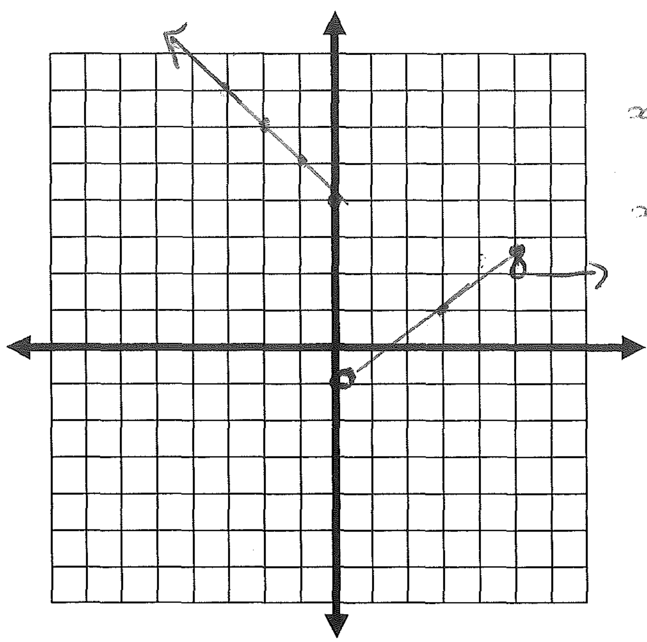
Function? Yes or No

$f(-2) =$ $-(-2)+4=6$

$f(0) =$ $-0+4=4$

$f(5) =$ $\dfrac{2(5)}{3}-1$

$=\dfrac{10}{3}-1=\dfrac{7}{3}$



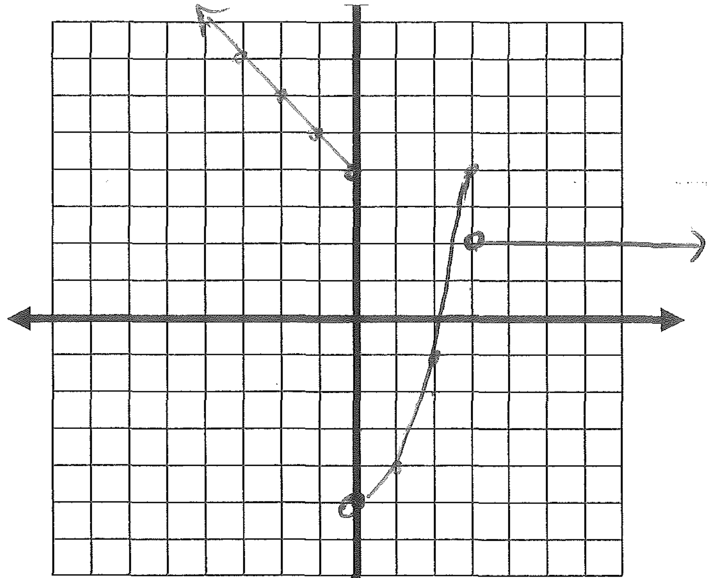
$x\to 5^-$ $y\to \frac{7}{3}$

$x\to 5^+$ $y\to 2$

$x\to 0^-$ $y\to 4$

$x\to 0^+$ $y\to -1$

4. $f(x)=\begin{cases} -x+4 & x\le 0 \\ x^2-5 & 0<x\le 3 \\ 2 & x>3 \end{cases}$

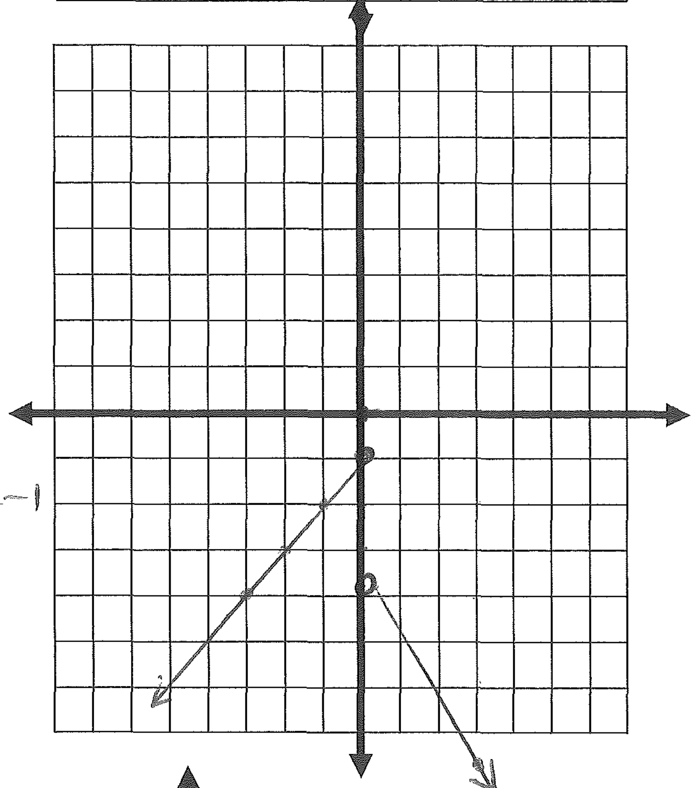


$f(-2)=$

$f(0)=$

$f(5)=$

5. $f(x)=\begin{cases} -|x-2|+1 & x\le 0 \\ -\dfrac{4x}{3}-4 & x>0 \end{cases}$

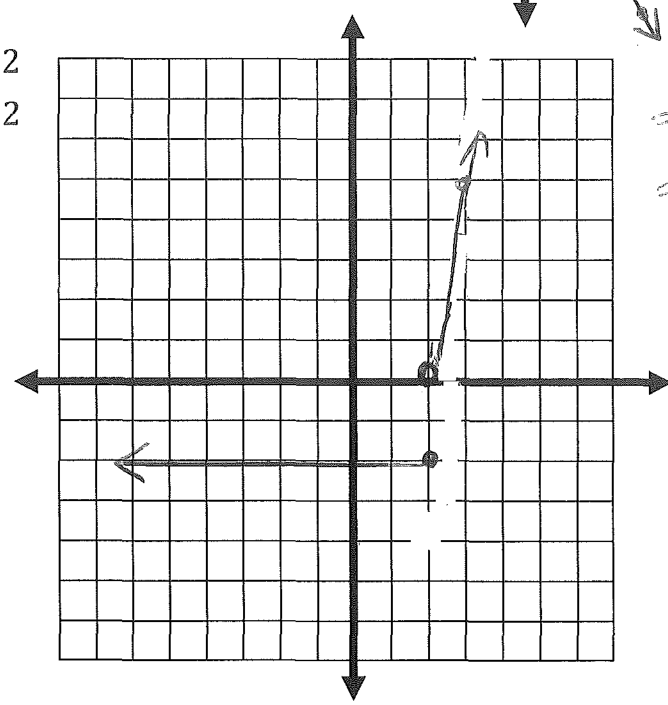


$f(-2)=$ $-|-2-2|+1=-3$

$f(0)=$ $-|0-2|+1=-2+1=-1$

$f(5)=$ $-\dfrac{4(5)}{3}-4=\dfrac{-32}{3}$

6. $f(x)=\begin{cases} -3 & x\le 2 \\ x^2-4 & x>2 \end{cases}$



$x\to 2^+$ $y\to 0$

$x=3$ $y=5$

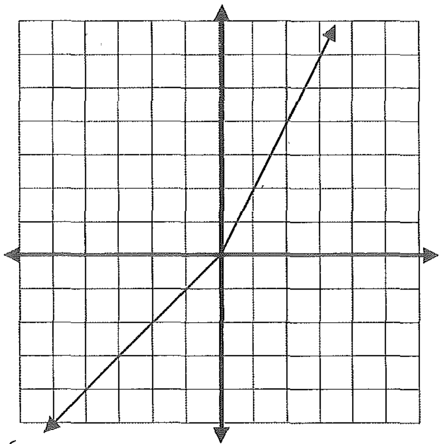
$f(-4)=$ -3

$f(0)=$ -3

$f(3)=$ 5

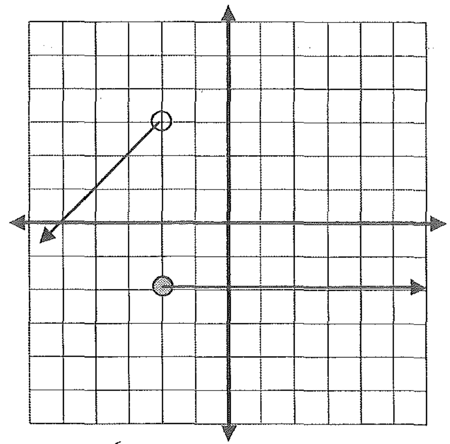
Part II. Write equations for the piecewise functions whose graphs are shown below. Assume $\square = 1$.

1.



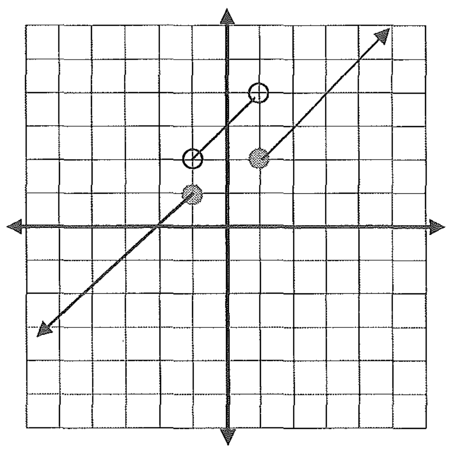
$$f(x) = \begin{cases} 2x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

2.



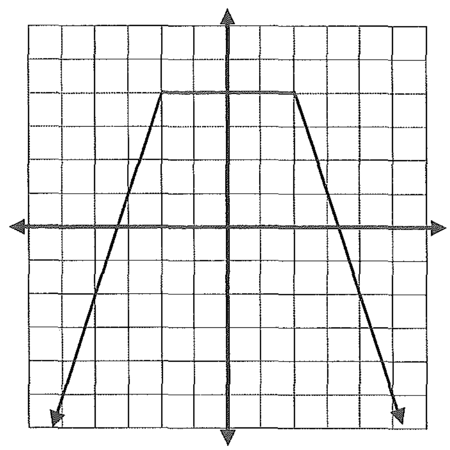
$$f(x) = \begin{cases} -2, & x \geq -2 \\ -x+5, & x < -2 \end{cases}$$

3.



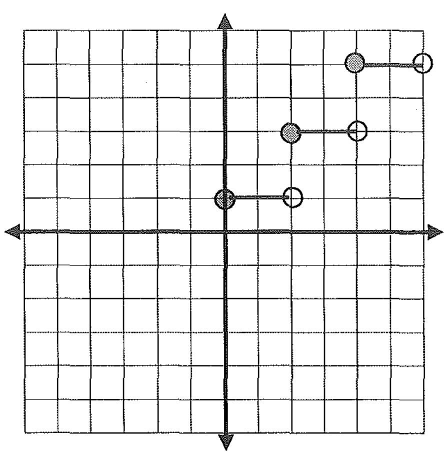
$$f(x) = \begin{cases} x+2, & x \leq -1 \\ x+3, & -1 < x < 1 \\ x+1, & x \geq 1 \end{cases}$$

4.



$$f(x) = \begin{cases} \\ \end{cases}$$

5.



$$f(x) = \begin{cases} 1, & 0 \leq x < 2 \\ 3, & 2 \leq x < 4 \\ 5, & 4 \leq x < 6 \end{cases}$$