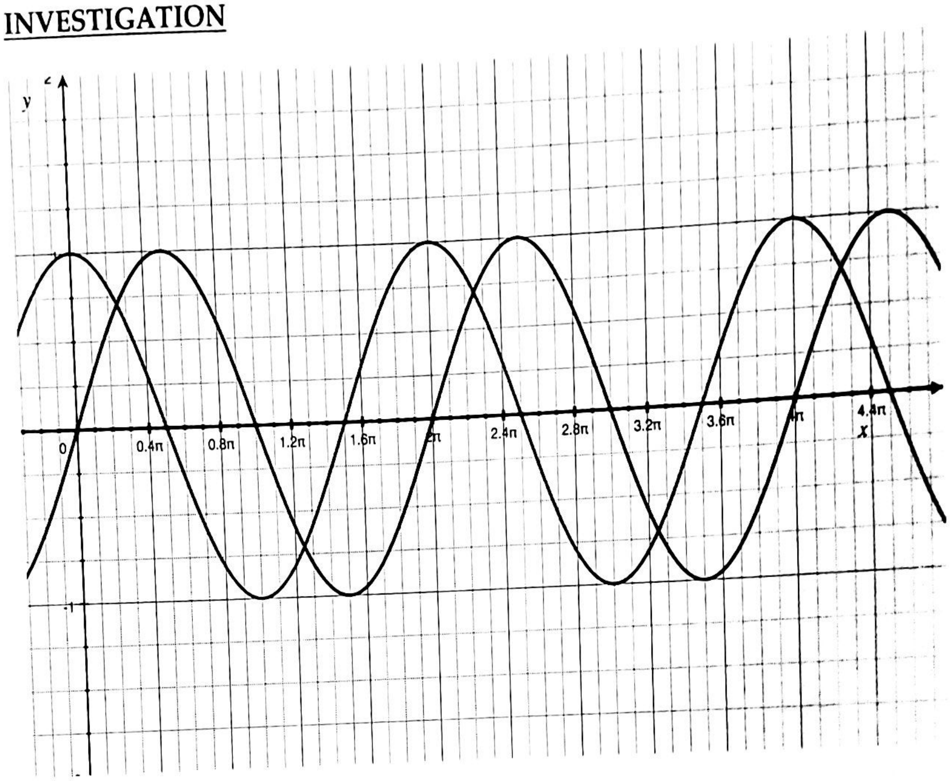
Day 5: 5.4 Derivatives of Sine and Cosine Functions

INVESTIGATION



SUMMARY:

$y = \sin(x)$ $\quad y' = \cos(x)$

$y = \cos(x)$ $\quad y' = -\sin(x)$

Example 1: Find the derivatives

a) $y = \sin(x^2)$

$y' = \cos(x^2) \cdot (2x)$

$= 2x \cos(x^2)$

b) $y = \cos^2(x)$

$y = [\cos x]^2$

$y' = 2[\cos x]^1 [\cos x]'$

$= 2(\cos x)(-\sin x)$

$= -2 \sin x \cos x$

$= -\sin 2x$

c) $y = \cos(2x^2 - 3x + 1)$

$$y' = -\sin(2x^2 - 3x + 1)\cdot(4x - 3)$$

d) $y = x\sin(x)$

$$y' = 1\,\sin x + x\cos x$$

e) $y = \frac{\sin(3x)}{x^2}$

$$y' = \frac{(3\cos 3x)(x^2) - (2x)\sin 3x}{x^4}$$

$$= \frac{(3\cos 3x)(x) - 2\sin 3x}{x^3}$$

e) $y = \sin[\cos(3x)]$

$$y' = \cos[\cos(3x)]\cdot[\cos(3x)]'$$

$$= \cos[\cos(3x)](-\sin(3x))\cdot 3$$

$$= -3\cos[\cos(3x)]\cdot\sin 3x$$

Example 2: Find the equation for the tangent to $f(x) = \sin(x + \frac{\pi}{2})$ at $x = \frac{\pi}{3}$

$$f'(x) = \cos\left(x + \frac{\pi}{2}\right)$$

$$f'\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \cos\left(\frac{5\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\left.\begin{array}{l} x = \frac{\pi}{3} \\ y = \frac{1}{2} \\ m = -\frac{\sqrt{3}}{2} \end{array}\right\} \quad \begin{array}{l} y - y_1 = m(x - x_1) \\ y - \frac{1}{2} = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) \end{array}$$

OR $\quad y = -\frac{\sqrt{3}}{2}x + \frac{\sqrt{3}\pi}{6} + \frac{1}{2}$

Example 3: Find the max and the min of $y = \sqrt{3}\,x + 2sinx$ on the interval $0 \le x \le \pi$

$y' = \sqrt{3} + 2\cos x$

NOTE: $150° = \frac{5\pi}{6}$, $210° = \frac{7\pi}{6}$

$y' = 0 \Rightarrow$ $2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$\therefore y(0) = \sqrt{3}(0) + 2\sin(0) = 0$

Q2: $x = 150° \in [0, \pi]$

$y\left(\frac{5\pi}{6}\right) = (\sqrt{3})\left(\frac{5\pi}{6}\right) + 2\sin\left(\frac{5\pi}{6}\right) \doteq 5.53$

Q3: $x = 210° \notin [0, \pi]$

$y(\pi) = (\sqrt{3})(\pi) + 2\sin(\pi)$

$= \sqrt{3}\pi \doteq 5.44$

$\therefore$ absolute max $= y\left(\frac{5\pi}{6}\right) \doteq 5.53$

absolute min $= y(0) = 0$

Example 4:

a) $y = \cos^5(x)$

$y = [\cos x]^5$

$y' = 5\cos^4 x\,(-\sin x)$

$= -5\sin x\cos^4 x$

b) $y = \cos(x^5)$

$y' = -\sin(x^5) \cdot 5x^4$

$= -5x^4\sin(x^5)$

c) $y = \cos(5x)$

$y' = -\sin(5x) \cdot 5$

$= -5\sin(5x)$

Example 5:

a) $y = \csc(x)$

$y = \frac{1}{\sin x} = [\sin x]^{-1}$

$y' = -1[\sin x]^{-2} \cdot (\cos x)$

$= -\frac{\cos x}{\sin^2 x}$

$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$

$= -\cot x\csc x$

b) $y = \sin\left(\frac{1}{x}\right)$

$y' = \cos\left(\frac{1}{x}\right)\left(\frac{1}{x}\right)'$

$= -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$