Day 4: 5.3 Optimization Problems Involving Exponential Functions

Example 1:
Find the absolute maximum and minimum of the function $f(x) = (x + 3)e^{-3x}$ on the interval $-4 \le x \le 3$

$f'(x) = e^{-3x} + (x+3)e^{-3x} \cdot (-3)$

$= e^{-3x}[1 + (x+3)(-3)]$

$= e^{-3x}[1 - 3x - 9]$

$= e^{-3x}(-3x-8)$

$-3x - 8 = 0 \Rightarrow x = \frac{-8}{3} \in [-4, 3]$.

$f(-4) = -e^{12}$ ← absolute min

$f(-8/3) = \frac{1}{3}e^{8} \approx 993.65$ → absolute max

$f(3) = 6e^{-9} \approx 0.0007$

Example 2:
A new motorcycle is purchased for $10000. The value of the motorcycle depreciates over time and is modeled by $V(t) = 10000e^{-t/4}$ where V is the value of the motorcycle after t years.
a) At what rate is the value depreciating the instant it is driven off the dealer's lot (t = 0)

need $V'(0)$

$V'(t) = 10000e^{-t/4} \cdot \left(-\frac{1}{4}\right)$

$V'(0) = 10000\left(-1/4\right)e^{0}$

$= -2500$

∴ The depreciation rate is $2500 /year the instant the motorcycle is driven off the dealer's lot.

b) At what rate the motorcycle is depreciating at the time it is one quarter of initial value?

initial $P = 10000$ quarter $= 2500$

∴ $2500 = 10000e^{-t/4}$

$\frac{1}{4} = e^{-t/4}$ take "ln" of both sides

$\ln\frac{1}{4} = \ln e^{-t/4}$

$\ln\frac{1}{4} = (\ln e)\left(-\frac{t}{4}\right)$

$-\frac{t}{4} = \ln 4^{-1}$

$-t = -4\ln 4$

$t = \ln 4^4 \approx 5.55$

$$\boxed{V'(\ln 4^4) = 10000\left(e^{-\frac{\ln 4^4}{4}}\right)\left(-\frac{1}{4}\right) = -625}$$

∴ The depreciation rate is $625/year when the motorcycle is worth $2500.

Example 3:

Suppose that the monthly revenue in thousands of dollars, for the sale of x hundred units of an electronic item is given by the function $R(x) = 40x^2e^{-0.4x} + 30$, where the maximum capacity of the plant is 800 units. Determine the number of units to produce in order to maximize revenue.

$R(x) = 40x^2 e^{-0.4x} + 30, \quad 0 \le x \le 8$

$R'(x) = (80x)e^{-0.4x} + (40x^2)(e^{-0.4x})(-0.4)$

$= 40x e^{-0.4x}[2 + (x)(-0.4)]$

$R'(x) = 0$ implies
$\begin{cases} e^{-0.4x} = 0 \Rightarrow \text{No soln} \\ 2 - 0.4x = 0 \Rightarrow 0.4x = 2 \\ \qquad x = 5 \in [0, 8] \end{cases}$

$\therefore \quad R(0) = 30$

$R(5) = 40(5)^2 e^{-0.4(5)} + 30 \approx 165.33$

$R(8) = 40(64)e^{-0.4(8)} + 30 \approx 134.35$

$\therefore$ 500 units must be produced to maximize the revenue.