Day 3: 5.2 Derivatives of the General Exponential Functions $y = b^x$

INVESTIGATION

$f(x) = 3^x$

$f(x+h) = 3^{(x+h)}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3^{(x+h)} - 3^x}{h}$$

$$= \lim_{h \to 0} \frac{3^x 3^h - 3^x}{h} = \lim_{h \to 0} \frac{3^x(3^h - 1)}{h}$$

$$= 3^x \lim_{h \to 0} \frac{3^h - 1}{h} = \left(\lim_{h \to 0} \frac{3^h - 1}{h} \right) 3^x$$

Note: The limit in the investigation approaches ln3. Hence, derivative of b^x involves lnb.

Derivative of $f(x) = b^x$

$$\boxed{f(x) = b^x, \qquad f'(x) = (\ln b)b^x.}$$

Derivative of composite functions involving b^x

$$\boxed{\text{if } f(x) = b^{g(x)} \text{ then } f'(x) = (\ln b)b^{g(x)}g'(x)}$$

Example1: Differentiate

a) $y = 7^{(x^2-5x)}$

$y' = \left(7^{x^2-5x}\right) \cdot (2x-5) \cdot \ln 7$

b) $f(x) = x^3 2^x$

$f'(x) = 3x^2 2^x + x^3 2^x \cdot \ln 2$

$= x^2 2^x (3 + x\ln 2)$

c) $g(x) = \sqrt{5^x} = (5^x)^{\frac{1}{2}} = 5^{\frac{1}{2}x}$

$g'(x) = 5^{\frac{1}{2}x} \cdot \ln 5 \cdot \frac{1}{2}$

Example 2: Determine the x-coordinate(s) of any local maximum or minimum values of the function $y = e^x - e^{2x}$

$y' = e^x - 2e^{2x}$ set $y' = 0$

$e^x(1 - 2e^x) = 0$

$e^x = 0$ OR $1 - 2e^x = 0$

No sol^n $\quad 2e^x = 1$

$e^x = \frac{1}{2} \Rightarrow x = \ln\frac{1}{2}$

$\ln 1/2$

y' $\quad +$ $\quad -$

$\therefore$ At $x = \ln\frac{1}{2}$, there will be a local maximum.

Example 3: Determine the equation of the tangent to $y = 2(3^x)$ at x = 2

$y' = 2(3)^x \cdot \ln 3$

$y'(2) = (2)(3)^2 \ln 3$

$= 18 \ln 3.$

$x = 2$

$y = 18$

$m = 18\ln 3$

$y - y_1 = m(x - x_1)$

$y - 18 = 18\ln 3(x - 2)$

OR $y = \underbrace{18\ln 3}_{m}\, x \underbrace{- 36\ln 3 + 18}_{b}.$

,8.