

Day 3: 5.2 Derivatives of the General Exponential Functions $y = b^x$

INVESTIGATION

$$f(x) = 3^x$$

$$f(x+h) = 3^{(x+h)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{3^{(x+h)} - 3^x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3^x 3^h - 3^x}{h} = \lim_{h \rightarrow 0} \frac{3^x (3^h - 1)}{h}$$

$$= 3^x \lim_{h \rightarrow 0} \frac{3^h - 1}{h} = \left(\lim_{h \rightarrow 0} \frac{3^h - 1}{h} \right) 3^x$$

Note: The limit in the investigation approaches $\ln 3$. Hence, derivative of b^x involves $\ln b$.

Derivative of $f(x) = b^x$

$$f(x) = b^x, \quad f'(x) = (\ln b)b^x.$$

Derivative of composite functions involving b^x

$$\text{if } f(x) = b^{g(x)} \text{ then } f'(x) = (\ln b)b^{g(x)}g'(x)$$

Example 1: Differentiate

a) $y = 7^{(x^2-5x)}$

$$y' = (7^{x^2-5x}) \cdot (2x-5) \cdot \ln 7$$

b) $f(x) = x^3 2^x$

$$f'(x) = 3x^2 2^x + x^3 2^x \cdot \ln 2$$

$$= x^2 2^x (3 + x \ln 2)$$

c) $g(x) = \sqrt{5^x} = (5^x)^{\frac{1}{2}} = 5^{\frac{1}{2}x}$

$$g'(x) = 5^{\frac{1}{2}x} \cdot \ln 5 \cdot \frac{1}{2}$$

Example 2: Determine the x-coordinate(s) of any local maximum or minimum values of the function

$y = e^x - e^{2x}$

$$y' = e^x - 2e^{2x} \quad \text{set } y' = 0$$

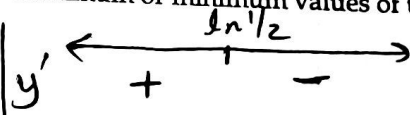
$$e^x(1 - 2e^x) = 0$$

$$e^x = 0 \quad \text{OR} \quad 1 - 2e^x = 0$$

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$$2e^x = 1$$

$$e^x = \frac{1}{2} \Rightarrow x = \ln \frac{1}{2}$$



\therefore At $x = \ln \frac{1}{2}$, there will be a local maximum.

Example 3: Determine the equation of the tangent to $y = 2(3^x)$ at $x = 2$

$$y' = 2(3)^x \cdot \ln 3$$

$$y'(2) = (2)(3)^e \ln 3$$

$$= 18 \ln 3.$$

$$x = 2 \quad \left\{ \begin{array}{l} y - y_1 = m(x - x_1) \end{array} \right.$$

$$y = 18 \quad \left\{ \begin{array}{l} y - 18 = 18 \ln 3 (x - 2) \end{array} \right.$$

$$m = 18 \ln 3$$

$$\text{OR } y = \underbrace{18 \ln 3}_m x - \underbrace{36 \ln 3 + 18}_b.$$