Day 1: Do you remember?

Properties of Exponents Laws (Power Laws – integer exponent)

$a^x \cdot a^y = a^{x+y}$

$a^{-x} = \frac{1}{a^x}$

$\frac{a^x}{a^y} = a^{x-y}$

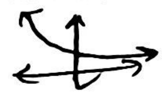
$\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$

$(a^x)^y = a^{xy}$

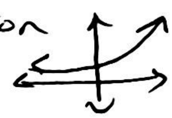
$a^0 = 1, \ a \neq 0$

Properties of the Exponential Function $y = b^x$, $b>0$, $b \neq 1$(real number exponent)

$0 < b < 1$: decreasing function



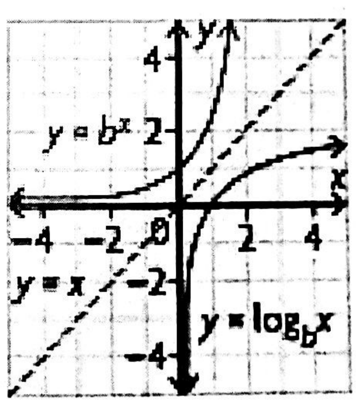
$b > 1$: increasing function



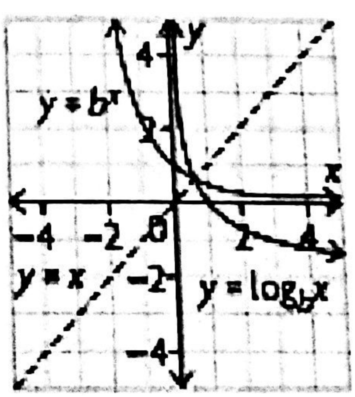
$(0, 1)$ is the y-int.

$y = 0$ is the horizontal asymptote.

Graphs of $y = \log_b x$ and $y = b^x$

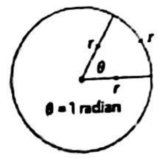


for $b > 1$

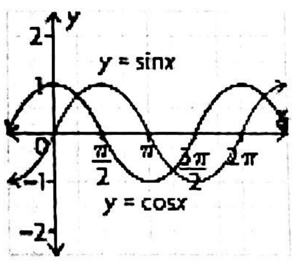


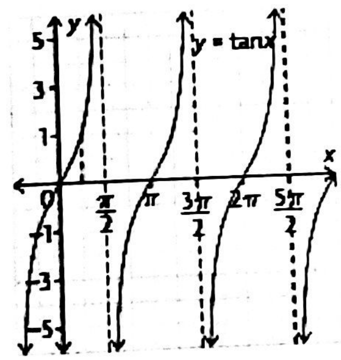
for $0 < b < 1$

Radian Measures



Sine, Cosine and Tangent Functions





Transformation of Sinusoidal Functions

*

Trigonometric Identities

Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Reflection Identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

Cofunction Identities

$\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$

* $y = a\sin[k(x-d)]+c$

$|a|$: amplitude

Equation of axis of curve : $y=c$

$max = c + |a|$ $\quad$ $min = c - |a|$

Period $= \dfrac{2\pi}{k}$

Extra practice solutions.

3 a) $e^{\ln 5} = 5$ since e and $\ln$ are inverse of eachother.

b) $\ln e^2 = 2 \ln e = 2(1) = 2$

c) $2 \ln e = 2(1) = 2$

d) $e^{5 \ln 2} = e^{\ln 2^5} = 2^5 = 32$

e) $\ln \sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2} \ln e = \frac{1}{2}$. (since $\ln e = 1$)

f) $\ln 2 + 2\ln 3 - \ln 18 = \ln 2 + \ln 3^2 - \ln 18$

$= \ln 2 + \ln 9 - \ln 18$

$= \ln \frac{(2)(9)}{18} = \ln 1 = 0$

4) a) $e^x = 4$

$\ln e^x = \ln 4$

$x(\ln e) = \ln 4$

$\boxed{\therefore x = \ln 4}$

b) $\ln x = 6$

$\boxed{e^6 = x}$

c) $\ln(2x-1) = 1$

$e^1 = 2x - 1$

$2x = e + 1$

$\boxed{x = \frac{e+1}{2}}$

d) $e^{3x+5} = 10$

$\ln e^{3x+5} = \ln 10$

$3x + 5 = \ln 10$

$3x = \ln 10 - 5$

$\boxed{x = \frac{\ln 10 - 5}{3}}$

e) $\ln e^{3-x} = 8$

$3 - x = 8$

$-x = 8 - 3$

$-x = 5$

$\boxed{x = -5}$

f) $\ln x = \ln 4 + \ln 7$

$\ln x = \ln(4 \times 7)$

$\ln x = \ln 28$

$\boxed{\therefore x = 28}$

g) $\ln(\ln x) = 2$

$\ln x = e^2$

$\boxed{x = e^{e^2}}$

h) $e^{e^x} = 5$

$\ln e^{e^x} = \ln 5$

$e^x = \ln 5 \Rightarrow \boxed{x = \ln(\ln 5)}$

Take ln of both sides.

5) a) $\ln(x+1) = 3$

$e^3 = x+1$

$\boxed{x = e^3 - 1 \approx 19.085537}$

5b) $e^{-x} = \frac{1}{2}$

$\ln e^{-x} = \ln 0.5$

$-x = \ln 0.5$

$\boxed{x = -\ln 0.5 \approx 0.693147}$

5c) $e^{5x+3} = 10$

Take "ln" of both sides

$\ln e^{5x+3} = \ln 10$

$5x+3 = \ln 10$

$\boxed{x = \frac{\ln(10) - 3}{5} \approx -0.139482}$

d) $2^{x-5} = 3$

$\ln 2^{x-5} = \ln 3$

$x-5 = \frac{\ln 3}{\ln 2}$

$\boxed{x = \frac{\ln 3}{\ln 2} + 5 \approx 6.584963}$

6) a) $\frac{1}{3}\ln x + 2\ln(3x-5)$

$= \ln x^{\frac{1}{3}} + \ln(3x-5)^2$

$= \ln(3x-5)^2(x)^{\frac{1}{3}}$

$= \ln[\sqrt[3]{x}\,(3x-5)^2]$

b) $2\ln x - \frac{1}{2}\ln(x^2-1) + 3\ln(x^2+1)$

$= \ln x^2 - \ln(x^2-1)^{\frac{1}{2}} + \ln(x^2+1)^3$

$= \ln x^2 + \ln(x^2+1)^3 - \ln\sqrt{x^2-1}$

$= \ln\left[\frac{(x^2)(x^2+1)^3}{\sqrt{x^2-1}}\right]$

$\ln a + \ln b - \ln c$

$= \ln\left(\frac{ab}{c}\right)$