Lesson 6.5 The Amount and Present Value of an Annuity

Goal: ***Calculate the amount (future value) of a simple ordinary annuity***
Calculate the present value of a simple ordinary annuity

- An ***annuity*** is a series of ***equal*** payments paid ***in to or out of*** an account at ***regular*** intervals
- In an ***ordinary simple annuity***, payments are made at the ***end*** of each ***compounding*** period
- The ***AMOUNT*** of an annuity (***future value***) is the sum of regular deposits plus ***interest***

Compound Periods ***(# times per year)***

Annually:	1	Semi-Annually:	2
Monthly:	12	Semi-Monthly:	24
Weekly:	52	Bi-weekly:	$\frac{52}{2}=26$
Quarterly:	4	Daily:	365

The ***AMOUNT*** of an ordinary simple annuity is given by the formula $A = \dfrac{R\left[(1+i)^n - 1\right]}{i}$, where

A = Final Amount
R = Payment (per compounding period)
i = interest rate (per compounding period)
n = number of compounding periods

This formula can only be used when the ***payment interval is the same as the compounding period***

Example Suppose $450 were deposited at the end of each quarter for 1.5 years into an annuity that earns 10% per year compounded quarterly

a) What is the amount of the annuity?

$A = ?$
$R = \$450$
$i = \frac{0.10}{4} = 0.025$
$n = 4 \times 1.5 = 6$

$$A = \frac{R[(1+i)^n - 1]}{i}$$
$$= \frac{450[(1+0.025)^6 - 1]}{0.025}$$
$$= 2874.48$$

∴ The annuity is worth $2874.48 in 1.5 years

The ***INTEREST*** of an ordinary simple annuity is given by the formula $I = A - Rn$, where I is interest amount

b) How much interest did the annuity earn?

We deposited $\$450 \times 6 = \$2700$
The account has $\$2874.48$.
∴ Interest $= \$2874.48 - \$2700$
$= \$174.48$
∴ The annuity earned $\$174.48$.

The ***present value*** of an annuity is the ***principal*** that must be invested ***TODAY*** to provide regular payments

The ***PRESENT VALUE*** of an ordinary simple annuity is given by the formula $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$, where

PV = Present value (amount invested today) i = interest rate (per compounding period)

R = Payments (per compounding period) n = number of compounding periods.

This formula can only be used when the ***payment interval is the same as the compounding period***

Example Victor wants to withdraw $700 at the end of each month for 8 months, starting 1 month from now. His bank account earns 5.4% per year compounded monthly.

a) How much must Victor deposit in his account TODAY to pay for the withdrawals?

$PV = ?$

$R = \$700$

$i = \frac{0.054}{12} = 0.0045$

$n = 8$

$PV = \frac{700\left[1-(1+0.0045)^{-8}\right]}{0.0045}$

$= \$5488.28$

∴ Victor must deposit $5488.28

The ***INTEREST*** of an ordinary simple annuity is given by the formula $I = Rn - PV$, where I is interest

b) How much interest did the annuity earn?

Victor will withdraw $\$700 \times 8 = \$5600$.

Victor deposited $5488.28

Interest = $\$5600 - \$5488.28 = \$111.72$

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