Lesson 5.9 Exponential Equations

Goal: Use equality of powers with a common base to solve exponential equations

An exponential equation is an equation that contains a variable in the exponent

Some exponential equations can be solved ***WITHOUT TECHNOLOGY*** by writing both sides of the equation as powers of the same base.

Equality of Powers with a Common Base *****Called Equating the Powers*****

If $a^m = a^n$, then $m = n$ $[a > 0, a \neq 1]$

EXAMPLE 1 Since 4^x and 4^3 are both powers of 4, the exponents must be equal, so the solution to $4^x = 4^3$ is $x = 3$

EXAMPLE 2 ***Finding a Common Base***

Solve each of the following.

a) $5^x = 5^6$

$x = 6$

b) $2^x = 32$

$2^x = 2^5$

$x = 5$

c) $2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

d) $2^x = \frac{1}{4}$

$2^x = 2^{-2}$

$x = -2$

e) $7^{3x-4} = 49$

$7^{3x-4} = 7^2$

$3x - 4 = 2$

$3x = 2 + 4$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

g) $3^{5x+8} = 27^{x}$

$3^{5x+8} = (3^{3})^{x}$

$3^{5x+8} = 3^{3x}$

$5x+8 = 3x$

$5x+8-3x=0$

$2x+8=0$

$2x=-8$

$x=-4$

STEPS
1. make bases the same
2. Simplify (remove brackets)
3. Set exponents equal.
4. Solve the equation using opposites

h) $4^{2(x-4)} = 8^{3x-2}$

$(2^{2})^{2(x-4)} = (2^{3})^{3x-2}$

$(2^{2})^{2x-8} = (2^{3})^{3x-2}$

$2^{4x-16} = 2^{9x-6}$

$4x-16 = 9x-6$

$4x-9x = -6+16$

$-5x = 10$

$x = \frac{10}{-5}$

$x=-2$

EXAMPLE 3 A cross country skier forgets a mug of coffee and a muffin in a snow bank. Their temperatures, in degrees Celsius, after t minutes can be modelled by the formulas: $T_{coffee} = 81\times3^{-2t}$ and $T_{muffin} = 27\times3^{-1}$. Determine when the coffee cools to the same temperature as the muffin.

$T_{coffee} = T_{muffin}$

$81\times3^{-2t} = 27\times3^{-1}$

$3^{4}\times3^{-2t} = 3^{3}\times3^{-1}$

$3^{4-2t} = 3^{3-1}$

$4-2t = 3-1$

$4-2t = 2$

$-2t = 2-4$

$-2t=-2$

$t=\frac{-2}{-2}$

$t=1$

* When multiplying powers with the same base, add exponents

Practice: Page 384 #1 – 3 (cd), 4, 5, 6, 7aceg, 10abd, 15, 16ab