Lesson 5.8 Rational Exponents

Goal: Explore the meaning of rational exponents

ACTIVITY – Exploring $a^{\frac{1}{n}}$

Examine the entries in the tables below. Determine the pattern to complete the next entries in each table.

Exponent 2	Exponent -2	Exponent $\frac{1}{2}$
$1^2 = 1$	$1^{-2} = 1$	$1^{\frac{1}{2}} = 1$
$2^2 = 4$	$2^{-2} = \frac{1}{4}$	$4^{\frac{1}{2}} = 2$
$3^2 = 9$	$3^{-2} = \frac{1}{9}$	$9^{\frac{1}{2}} = 3$
$4^2 = 16$	$4^{-2} = \frac{1}{16}$	$16^{\frac{1}{2}} = 4$
$5^2 = 25$	$5^{-2} = \frac{1}{25}$	$25^{1/2} = 5$
$6^2 = 36$	$6^{-2} = \frac{1}{36}$	$36^{1/2} = 6$
$7^2 = 49$	$7^{-2} = \frac{1}{49}$	$49^{1/2} = 7$

Exponent 3	Exponent -3	Exponent $\frac{1}{3}$
$1^3 = 1$	$1^{-3} = 1$	$1^{\frac{1}{3}} = 1$
$2^3 = 8$	$2^{-3} = \frac{1}{8}$	$8^{\frac{1}{3}} = 2$
$3^3 = 27$	$3^{-3} = \frac{1}{27}$	$27^{\frac{1}{3}} = 3$
$4^3 = 64$	$4^{-3} = \frac{1}{64}$	$64^{\frac{1}{3}} = 4$
$5^3 = 125$	$5^{-3} = \frac{1}{125}$	$125^{1/3} = 5$
$6^3 = 216$	$6^{-3} = \frac{1}{216}$	$216^{1/3} = 6$
$7^3 = 343$	$7^{-3} = \frac{1}{343}$	$343^{1/3} = 7$

Compare the entries in the first and second column of each table. Describe the relationship that you see.

The second column is the reciprocal (flip) of the first column. A negative exponent flips the numerator and denominator

Compare the entries in the first and third column. What do you think it means to raise a number to an exponent of ½ or ⅓?

Raising a number to $1/2$ is the same as square rooting
Raising a number to $\frac{1}{3}$ is the same as cube root.

Use your results above to define a formula for $\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

SAME! Just different notation

ACTIVITY *– Exploring* $a^{\frac{m}{n}}$

Examine the entries in the tables below. Use your calculator to complete each table.

To do a fractional (rational) exponent on your calculator you will need to:

- Use exponent button on your calculator (either the x^y, y^x, or ^ button)
- Use brackets around the fraction
- For example: Enter $25^{\frac{3}{2}}$ as **25** x^y **(3 ÷ 2) =** → 25 ^ (3 ÷ 2)

a	$a^{\frac{1}{2}}$	$a^{\frac{3}{2}}$	$a^{\frac{5}{2}}$
1	→	1	1
4	→	8	32
9	→		243
16	→	64	1024

a	$a^{\frac{1}{3}}$	$a^{\frac{2}{3}}$	$a^{\frac{5}{3}}$
		1	1
		4	32
		9	243
		16.	1024

Compare the entries in the second, third, and fourth columns of each table.

How do the values of $a^{\frac{3}{2}}$ and $a^{\frac{5}{2}}$ relate to the values of $a^{\frac{1}{2}}$?

$a^{\frac{3}{2}}$ is the cube of $a^{\frac{1}{2}}$

$a^{\frac{5}{2}}$ is $a^{\frac{1}{2}}$ raised to the power of 5

How do the values of $a^{\frac{2}{3}}$ and $a^{\frac{5}{3}}$ relate to the values of $a^{\frac{1}{3}}$?

$a^{2/3}$ is the square of $a^{\frac{1}{3}}$

$a^{\frac{5}{3}}$ is $a^{\frac{1}{3}}$ raised to the power of 5

Use your results above to define a formula for $\boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$

(n is the root.)

Summary: ***Conditions***

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$, m and n are integers
n is positive.
if n is even, $a \geq 0$

- ***Radical*** means there is a root $\sqrt{\ }$
- ***Rational*** means there is an exponent in fraction form

EXAMPLE 1 Rewrite each expression using rational exponents.

a) $\sqrt[2]{25} = 25^{\frac{1}{2}}$ **b)** $\sqrt[3]{-125} = (-125)^{\frac{1}{3}}$ **c)** $\sqrt[4]{1.05^3}$ $(1.05)^{\frac{3}{4}}$

EXAMPLE 2 Rewrite each expression in radical form and <u>then evaluate</u>.

a) $81^{\frac{1}{2}} = \sqrt{81}$ $= 9$ **b)** $(-64)^{\frac{1}{3}} = \sqrt[3]{-64}$ $= -4$ **c)** $32^{\frac{4}{5}} = \sqrt[5]{32^4}$ $= 2^4$ $= 16$

EXAMPLE 3 Solve for the unknown variable, x.

a) $x^4 = 16$
$x = \sqrt[4]{16}$
$x = 2$

b) $x^{\frac{3}{2}} = -27$
$\sqrt{x^3} = -27$
$x^3 = (-27)^2$
$x = \sqrt[3]{(-27)^2}$
$x = (-3)^2 = 9$

c) $x^{\frac{2}{3}} = 64$
$\sqrt[3]{x^2} = 64^3$
$x^2 = 64^3$
$x = \sqrt{64^3}$
$x = 8^3$
$x = 512.$

EXAMPLE 4 Under annual compounding, an initial investment of \$700 grows to \$900 in 5 years. Determine the <u>annual</u> interest rate, i, using the compound interest formula $A = P(1 + i)^n$.

$A = P(1+i)^n$
$900 = 700(1+i)^5$
$\frac{900}{700} = (1+i)^5$
$\frac{9}{7} = (1+i)^5$
$\sqrt[5]{\frac{9}{7}} = 1+i$

$\sqrt[5]{\frac{9}{7}} - 1 = i$
$0.051 = i$

Practice: Page 369 #2bde, 3 – 5 Page 376 #3ad, 5cd, 6acf, 9, 10ade, 12b, 13, 14ac