Lesson 5.6 Working with Formulas

Goal: ***Rearrange equations to rewrite the formula in terms of a different variable***
Plan and organize solutions for real life problems

A ***formula*** is a mathematical equation that relates two or more variables.

- ***P = 25T – 800*** might give the profit from ticket sales, where ***P*** is the **profit** and ***T*** is the **# of tickets sold**
- ***E = mc²*** (Einstein's theory of relativity) relates **Energy** to the **Mass** of an object and the **Speed of light**

EXAMPLE 1 The formula ***S = 0.6T + 331.5*** gives the approximate speed of sound in air, ***S*** metres per second, when the temperature is ***T*** degrees Celsius. Determine the speed of sound at -40 °C.

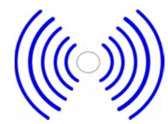
Formula → $S = 0.6T + 331.5$

Substitute for T → $S = 0.6(-40) + 331.5$

Solve for s → $S = -24 + 331.5$

$S = 307.5$

Sentence → ∴ The speed of sound is 307.5 m/s at −40°C



Sometimes we need to solve for a variable that isn't already by itself. To do this we rearrange the equations using an ***INVERSE OPERATIONS*** method. This means you "undo" or "reverse" what is in front of you. Order of operations (BEDMAS) are performed in **reverse** order.

B
E
D/M
A/S

EXAMPLE 2 Get P on one side of the equal sign

For each example, isolate variable P.

$A = P + I$
$A - I = P$
$P = A - I$

$\frac{A}{2} = \frac{2P}{2}$
$\frac{A}{2} = P$
$P = \frac{A}{2}$

$\sqrt{A} = \sqrt{P^2}$
$\sqrt{A} = P$
$P = \sqrt{A}$

$A - I = 2P^2 + I - I$
$\frac{A-I}{2} = \frac{2P^2}{2}$
$\sqrt{\frac{A-I}{2}} = \sqrt{P^2}$
$\sqrt{\frac{A-I}{2}} = P$
$P = \sqrt{\frac{A-I}{2}}$

For each example, isolate variable B.

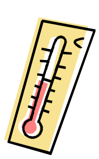
$\frac{C}{2} = \frac{2(B-N)}{2}$
$\frac{C}{2} = B - N$
$\frac{C}{2} + N = B$
$B = \frac{C}{2} + N$

$C = (\frac{B}{2} - N)$
$C = \frac{B}{2} - N$
$2(C+N) = \frac{B}{2} \times 2$
$2(C+N) = B$
$B = 2(C+N)$

$\sqrt{C} = \sqrt{(B-N)^2}$
$\sqrt{C} = B - N$
$\sqrt{C} + N = B$
$B = \sqrt{C} + N$

$\frac{C}{2} = \frac{2(\frac{B}{2} - N)^2}{2}$
$\frac{C}{2} = (\frac{B}{2} - N)^2$
$\sqrt{\frac{C}{2}} = \frac{B}{2} - N$
$\sqrt{\frac{C}{2}} - N = \frac{B}{2}$
$2(\sqrt{\frac{C}{2}} - N) = B$
$B = 2(\sqrt{\frac{C}{2}} - N)$

EXAMPLE 3 To convert from Celsius (C) to Fahrenheit (F), the formula $C = \frac{5(F-32)}{9}$ is used.



Determine a formula to convert from Fahrenheit to Celsius by isolating F.

$C = \frac{5(F-32)}{9}$

What if F = 10°F?
What is C?

$C = \frac{5(10-32)}{9}$) Subtract 32
$= \frac{5(-22)}{9}$) multiply by 5
$= \frac{-110}{9}$) divided 9.
$= -12.222$

$9C = \frac{5(F-32)}{9} \times 9$

$\frac{9C}{5} = \frac{5(F-32)}{5}$

$\frac{9C}{5} = F - 32$

$\frac{9C}{5} + 32 = F$

$F = \frac{9C}{5} + 32$

Solving Multi-Step Problems – Plan and organize your solution

- ***PLAN*** your solution by working backward from what you are trying to find to what you are given
  - Determine what numerical info is given and what you need to find
  - Decide what formula(s) to use
- ***WRITE*** the solution by working forward from what you are given to what you are trying to find
  - Convert quantities to similar units (if necessary)
  - Substitute known values (given or calculated) to solve for the unknown

EXAMPLE 4 – CHOOSING FORMULAS & CONVERTING MEASURES

A landscaper uses a bucket with radius 18 cm and height 18 cm to pour soil into a rectangular planter measuring 1.2 m by 40 cm by 20 cm. How many buckets of soil are needed to fill the planter?

Convert measures →

Planter: $1.2\,m \times 100\frac{cm}{m} = 120\,cm$
(Rectangle) 40 cm
20 cm

Bucket: $r = 18\,cm$
(Cylinder) $h = 18\,cm$

Planter | Bucket

Decide on formulas to use →
(VOLUME)

$V = L \times W \times H$ | $V = \pi r^2 h$

Substitute given values →

$= (120cm)(40cm)(20cm)$ | $= \pi(18)^2(18)$

Solve the problem →

$= 96000\,cm^3$ | $= 18\,321.77\,cm^3$

$\#\text{ buckets needed} = \frac{96\,000\,cm^3}{18\,321.77\,cm^3}$

$= 5.24$ times

Write a final statement →

∴ 5.24 buckets of soil are needed to fill the planter.

EXAMPLE 5 The area, A, of a circle with radius r is given by $A = \pi r^2$. Use the formula to determine the radius of a circular oil spill that covers an area of $5.0\,km^2$.

EXAMPLE 6 – MULTI-STEP PROBLEMS

A landscaper wants to estimate the cost of fertilizing a triangular lawn with side lengths 150 m, 200 m, and 300 m. One bag of fertilizer costs $19.98 and covers an area of 900 m^2. She uses Heron's formula to determine the area of the lawn: The area A of a triangle with side lengths a, b, and c, is given by $A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{a+b+c}{2}$. Estimate the cost to fertilize the lawn.

PLAN the solution: Work backwards

To find the _________________________________ I need to know the _____________________________

To find the ***# of bags needed*** I need to know the _______________________________

To find the ***area of the lawn*** I need to use the _______________________________

To use the ***formula for area*** I need to know the _______________________________

To find the ***value of s*** I need to know the _______________________________

WRITE the solution: Work forwards

Find s:

Find A:

Find # bags needed:

Find cost:

Practice: Page 346 #6 – 9; Page 354 #2, 4, 9, 11, 12, 14 – 16

$\boxed{\text{Isolate } y}$

$$x \overset{+3}{=} \frac{(y^2+8)}{12} \cancel{-3}^{+3}$$

$$12(x+3) = \frac{(y^2+8)}{\cancel{12}} \times \cancel{12}$$

$$12(x+3) \overset{-8}{=} y^2 \cancel{+8} \cancel{-8}$$

$$\sqrt{12(x+3)-8} = \sqrt{y^2}$$

$$\sqrt{12(x+3)-8} = y$$

$$y = \sqrt{12(x+3)-8}$$

What if $y=4$.
What is x?

$$x = \frac{(4^2+8)}{12} - 3$$ Square

$$x = \frac{(16+8)}{12} - 3$$ Add 8

$$x = \frac{24}{12} - 3$$ Divide by 12

$$x = 2 - 3$$ subtract 3

$$x = -1$$