Lesson 5.3 Exponential Models

Goal: *Apply exponential models to analyze and predict behaviour of real-world situations*

Exponential Models

Represent quantities that change at a constant percent rate rate (quantities are multiplied. by a fixed amount at regular intervals.

- In a table of values, the growth / decay factors are equal
- The graph resembles an exponential curve. curve
- The equation is written in the form $y = ab^x$ where a is the initial value and b is the growth/decay factor. Notice that the exponent is the x value

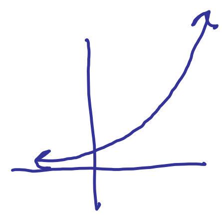
Growth/Decay Factors

In an exponential equation, $y = ab^x$, the growth/decay factor is given by the value of b

- If $b > 1$, the relation is increasing
- Growth factor = 1 + growth rate
- Growth rate = growth factor – 1

- If $0 < b < 1$, the relation is decreasing
- Decay factor = 1 – decay rate
- Decay rate = 1 – decay factor

Growth



Decay

EXAMPLE 1 Determine the growth or decay rate in each of the following:

a) $A = 500(1.071)^n$

Initial value

Growth Factor

Growth rate = 0.071

b) $P = 500(0.92)^t$

Decay Factor

Decay rate = 0.08

EXAMPLE 2 Which models represent exponential relations?

a)

t	A
0	35
1	25
2	15
3	5

$\frac{25}{35} = 0.714$

$\frac{15}{25} = 0.6$

$\frac{5}{15} = 0.333$

∴ Not exponential.

b)

d	P
0	51.2
1	64
2	80
3	100

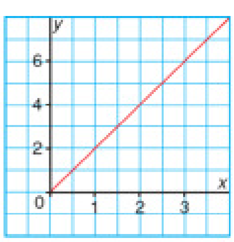
$\frac{64}{51.2} = 1.25$

$\frac{80}{64} = 1.25$

$\frac{100}{80} = 1.25$

∴ Exponential.

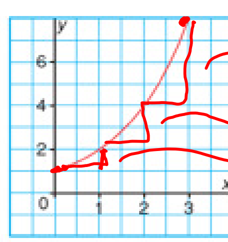
c)



Straight Line

∴ Not Exponential.

d)



4 ×2

2 ×2

1

∴ Exponential.

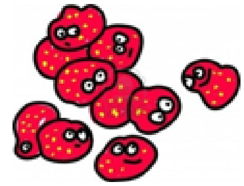
e) $y = 10(2)^x$ x in the exponent

∴ Exponential.

f) $y = 10x^2$ Exponent 2

∴ Not Exponential (Quadratic)

Comparing Pairs of Exponential Relations



Compare the initial value and compare the growth/decay factor

EXAMPLE 3 The equations of two colonies of bacteria are shown below.

Colony A: $P = 100(2)^t$ ***Colony B***: $P = 100(3)^t$

a) Use a ***graphing calculator*** to graph both equations. You will need to change window settings as shown (press WINDOW). Describe how the two graphs compare.

The graph of Colony B increases at a faster rate. (They start at the same point)

b) How would the graph for Colony A change if there were 200 bacteria initially?

$P = 200(2)^t$

- The graph moves up 100 units
- The rate of increase stays the same.

Fitting an Exponential Model to Data

We can use exponential regression to model data that appear lie along an exponential curve and produce a curve of best fit

EXAMPLE

Year	1921	1931	1941	1951	1961	1971	1981	1991	2001
B.C. Population (millions)	0.52	0.69	0.82	1.17	1.63	2.18	2.82	3.37	4.08

4

a) Use a ***graphing calculator*** to determine the exponential relation $y = ab^x$ that best fits the data above, where x is the number of years **since 1921** and y is the population of British Columbia in millions.

$y = 2.222033 \cdot 10^{-23}(1.02718)^x$

b) What do the values of a and b represent in this situation?

a is the initial value
b is the growth rate.

c) Estimate the population of British Columbia in 1985.

$x = 1985$

$y = 2.222033 \cdot 10^{-23}(1.02718)^{1985} = 2.92$ million people

Practice: Page 315 #2 – 7, *8, *11, *12 **technology needed for questions marked with an asterisk*