Lesson 5.2 – Linear and Quadratic Models

Goal: ***Apply linear models to analyze and predict behaviour of real-world situations***
Apply quadratic models to analyze and predict behaviour of real-world situations

Tables, ***graphs***, and ***equations*** are all examples of mathematical models

Linear Models

Represent quantities that increase or decrease by a constant amount over constant intervals

- In a table of values, the first differences are equal
- The graph is a Straight line
- The equation of the line can be written in the form $y=mx+b$, where m is the slope and b is the vertical intercept (y-intercept)
- The rate of change is $\frac{\text{rise}}{\text{run}}$ or $\frac{y_2-y_1}{x_2-x_1}$

EXAMPLE 1 Which models represent linear relations?

a)

Time (s)	Height (m)
0	60
1	55
2	40
3	15

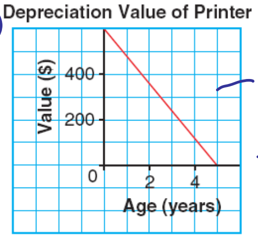
$55-60=-5$
$40-55=-15$
$15-40=-25$
$\therefore$ Non-Linear

b)

Time (h)	Earnings ($)
0	0
5	40
10	80
15	120

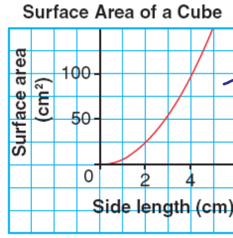
$40-0=40$
$80-40=40$
$120-80=40$
Linear!

c) Depreciation Value of Printer



straight line
$\therefore$ Linear

d) Surface Area of a Cube



curve
$\therefore$ non-linear

e) $y=2x+5$ $\therefore$ Linear
$(y=mx+b)$

f) $y=x^2+5$ exponent of 2,
$\therefore$ non-linear

Analyzing Graphs of Linear Relations

In real world graphs of linear relations:

- The vertical intercept represents the initial value value of the ***dependent*** variable
- The slope represents the rate of change in the ***dependent*** variable with respect to the ***independent*** variable

EXAMPLE 2 A cup of coffee is reheated in a microwave. The temperature, C degrees Celsius, of the coffee after t seconds can be modelled by the following linear equations. Explain what the numbers in the equations represent. How do the two equations compare to each other?

① ***500 W microwave:*** $C = 0.5t + 20$ ② ***1000 W microwave:*** $C = t + 20$

① The cup of coffee is 20°C before heating. The microwave heats it 0.5°C every second.

② The cup of coffee is 20°C before heating. The microwave heats it 1°C every second.

Compare: The coffees start at the same temperature. The 1000W microwave heats the coffee twice as fast as the 500W microwave.

Quadratic Models

Represent quantities that are non-linear which do not have a constant rate of change

- In a table of values, the second differences are equal
- The graph is a curve called a parabola
- The equation has a degree of 2 and is written in the form $y = ax^2 + bx + c$ where $a \neq 0$

EXAMPLE 3 Which models represent quadratic relations?

g)

h	*p*
0	250
1	238
2	202
3	142

238-250 = -12
202-238 = -36
142-202 = -60

-36-(-12) = -24
-60-(-36) = -24

∴ Quadratic

h)

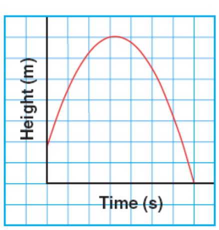
r	*Q*
0	32
1	48
2	72
3	108

48-32 = 16
72-48 = 24
108-72 = 36

24-16 = 8
36-24 = 12

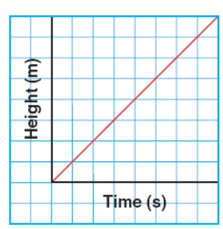
∴ Not Quadratic

i)



Quadratic

j)



Not Quadratic (Linear)

k) $y = x^2 + 7$ ← degree 2. ∴ Quadratic

l) $y = 3x + 2$ ← degree 1 ∴ Not Quadratic (linear)

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