Lesson 5.10 Applications of Exponential Equations

Goal: Model and solve real world applications in exponential growth and decay

Exponential relations are modelled by the exponential equation $y=ab^x$, where

a is the initial value

b is the rate of change.

$b > 1$ models growth

$0 < b < 1$ models decay

Real-world applications of exponential growth or decay may require solving the equation $y = ab^x$ for x

EXAMPLE 1*: *Guess & Check

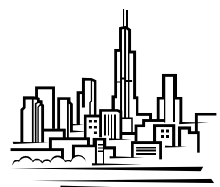
Ontario's population can be modelled by the equation $P = 9.4(1.0125)^t$, where *P* represents the population in millions *t* years after 1985. In which year did the population first exceed 10 million?

Beginning population: 9.4 million Ending population: 10 million.

Population is growing at a rate of: 1.0125 (1.25% increase)

Sub in the info given:

$P = 9.4(1.0125)^t$



$10 = 9.4(1.0125)^t$

Simplify:

$\frac{10}{9.4} = 1.0125^t$

$1.064 = 1.0125^t$

What value of t will make $1.0125^t = 1.064$

Use a ***TABLE OF VALUES*** to ***GUESS & CHECK*** a solution

t	1.0125^t
2	1.025
4	1.0509
5	1.064

→ The population first exceeded 10 million in 1990 (5 years after 1985)

EXAMPLE 2: Using a Graph

Redo example 1, but this time find the point of intersection in order to solve.

- Graph the exponential equation
- Graph the line P = 10 000 000
- Where do they meet?

TWO SPECIAL EXPONENTIAL GROWTH & DECAY EQUATIONS

$A = A_0(0.5)^{t/h}$

DOUBLING: $\boxed{A = A_0(2)^{t/d}}$

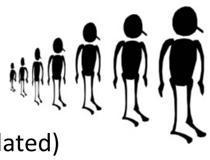
HALF-LIFE: $\boxed{A = A_0\left(\frac{1}{2}\right)^{t/h}}$

A_0 = Initial Amount

A = Final Amount

t = time (measured or calculated)

d = doubling time

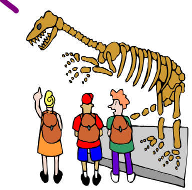


A_0 = Initial Amount

A = Final Amount

t = time (measured or calculated)

h = half-life



EXAMPLE 3: ***Solving an Application Involving Doubling Time***

A bacteria culture doubles in size approximately every 14 hours. Suppose this bacteria culture started with 100 individual bacteria. How long will it take for the bacteria population to reach 1000 individuals? Give your answer to one decimal place.

$A = 100(2)^{t/14}$

$1000 = 100(2)^{t/14}$

$\frac{1000}{100} = 2^{t/14}$

$10 = 2^{t/14}$

t	$2^{t/14}$
1	1.05
20	2.69
40	7.25
50	11.888
46	9.75
47	10.25
46.5	9.9965

∴ It will take 46.5 hours

EXAMPLE 4: ***Solving Applications Involving Half-Life***

Caffeine has a half-life of approximately 5 hours. Suppose you drink a cup of coffee that contains 200 mg of caffeine. How long will it take until there is less than 10 mg of caffeine left in your bloodstream?

decay

$A = A_0(0.5)^{t/h}$

$10 = 200(0.5)^{t/5}$

$\frac{10}{200} = (0.5)^{t/5}$

$0.05 = (0.5)^{t/5}$

t	$(0.5)^{t/5}$
10	0.25
20	0.0625
25	0.03125 ← Too small
22	0.04736
21	0.0544

∴ It will take 22 h until there is less than 10 mg in your bloodstream

Practice: Page 391 #1ac, 3ac, 8, 11b, 12bc, 14, 17, 21