Lesson 2.6 – Optimizing Volume and Surface Area

Goal: Determine the optimal volume and surface area for 3-dimensional figures

Optimization: The process of finding the most efficient use of available materials within given constraints.

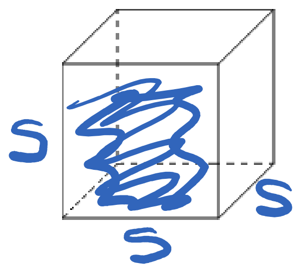
Key Concepts

1. Among all rectangular prisms with a given surface area, a Cube has the ***maximum*** volume
2. Among all rectangular prisms with a given volume, a Cube has the ***minimum*** surface area

FORMULAS for CUBE

Volume:

$V = s^3$



Surface Area:

$SA = 6s^2$

EXAMPLE 1

What dimensions produce a minimum surface area of a rectangular prism with a volume of 1000 cm^3?

Minimize Surface area $\Rightarrow$ Cube.

$V = 1000cm^2$

$V = s^3$

$1000 = s^3$

$\sqrt[3]{1000} = s$

$\rightarrow s = 10\,cm$

$\therefore$ The dimensions are 10cm by 10cm by 10cm.

EXAMPLE 2

What dimensions of a rectangular prism will produce a maximum volume if the surface area is 486 cm^2?

max volume $\Rightarrow$ cube.

$SA = 6s^2$

$\frac{486}{6} = \frac{\cancel{6}s^2}{\cancel{6}}$

$\sqrt{81} = \sqrt{s^2}$

$9 = s$

$\therefore$ The dimensions are all 9cm.

Optimizing with Restrictions

There may be constraints on the prism you are optimizing.

- The dimensions may have to be whole numbers or be multiples of a given number.
- Sometimes one or more of the sides of the object are missing or bordered by some physical barrier.

In these cases, the optimal rectangular prism will not be a cube. You can use diagrams or a table and graph to find the dimensions of the optimal rectangular prism.

EXAMPLE 3

Jacob is designing a glass candle holder. It will be a square-based rectangular prism with outer surface area of 225 cm^2, and no top.
Determine the maximum volume of the candle holder to the nearest cm^3. What are the dimensions of the candle holder?

$L \times L \times H =$

$SA = 225 cm^2$

$SA = 2lw + 2lh + 2wh$

Square base: $L = w$

$SA = 2l \cdot l + 2lh + 2lh$

$SA = 2l^2 + 4lh$

$225 = 2l^2 + 4lh$

$\frac{225 - 2l^2}{4l} = \frac{4lh}{4l}$

$h = \frac{225 - 2l^2}{4l}$

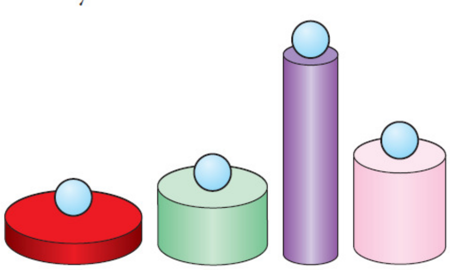
Base Side Length (cm)	Height of Prism (cm)	Volume (cm^3)	Surface Area (cm^2)
1	55.75	55.75	225
2	27.125	108.5	225
3	17.25	155.25	225
4	12.0625	193	225
5	8.75	218.75	225
6	6.375	229.5	225
7	4.536	222.264	225
8	3.031		225
9	1.75		225
10	0.625		225

∴ The dimensions are approximately 6cm by 6cm by 6.4cm

The area is 229.5cm^3.

EXAMPLE 4

Each cylindrical container has the same surface area.



Without measuring, order these containers from maximum to minimum volume. Explain your reasoning.

H, F, E, G.

Practice: Page 110 #1-5, 8, 9, 12-16