Lesson 2.5 – Optimizing Areas & Perimeters

Goal: Determine the optimal perimeter and area for 2-dimensional figures

Optimization: The process of finding the most efficient use of available materials within given constraints.

Key Concepts

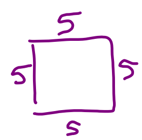
1. Among all rectangles with a given perimeter, a <u>square</u> has the ***maximum*** area
2. Among all rectangles with a given area, a <u>square</u> has the ***minimum*** perimeter

EXAMPLE 1

What dimensions produce an optimal area of a rectangle with perimeter 20 m? What is the maximum area?

$\text{length} = \frac{20m}{4}$

$= 5m$

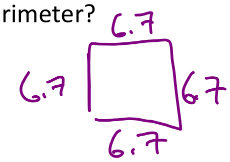


$\text{Area} = 5 \times 5$

$= 25 m^2.$

EXAMPLE 2

What dimensions produce an optimal perimeter of a rectangle with area 45 m^2? What is the minimum perimeter?



$\text{length} = \sqrt{45 m^2}$

$= 6.7 m.$

$\text{Perimeter} = 4 \times 6.7$

$= 26.8 m$

Optimizing with Restrictions

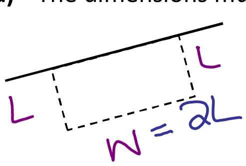
It may not be possible to form a square because of certain restrictions. Restrictions such as:

- The length and width need to be whole numbers
- One or more sides are enclosed by natural boundaries (house, pond, etc.)

EXAMPLE 3

A rectangular garden is to be fenced using the wall of a house as one side of the garden. The garden should have an area of 40 m^2. Determine the minimum perimeter and dimensions of the garden if:

a) The dimensions must be whole meters



$L \times W = 40 m^2$

$W = \frac{40}{L}$

→ twice → once

l	*w*	*P*
1	40	42
2	20	24
4	10	18
5	8	18
8	5	21
10	4	24
20	2	42
40	1	81

$P = 2l + w.$

b) The dimensions can be decimals

L	W	P
4.2	9.52	17.92
4.4	9.09	17.89
4.6	8.70	17.9
4.8	8.33	17.93

$A = L \cdot W$

$A = L(2L)$

$A = 2L^2$

$40 = 2L^2$

$20 = L^2$

$\sqrt{20} = L$

$L = 4.47 m.$

$W = 2 \times 4.47$

$= 8.94$

$P = 2(4.47) + 8.94$

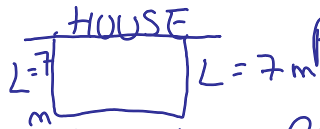
$= 17.88 m.$

When there are restrictions such as the above and a square cannot be formed the optimal shape will occur when the: Total length = Total width

* Rectangle with 3 sides → $W = 2L$

EXAMPLE 4

Ngaio has 28 m of fencing to build a pen for her dog. She plans to build the pen along one wall of her house as shown? What are the dimensions of the pen with the greatest area?



$P = 28m$

$W = 2L$
$= 2 \times 7$
$= 14m$

$P = 2L + W$
$P = 2L + 2L$
$P = 4L$
$\frac{28}{4} = \frac{4L}{4}$
$7m = L$

$Area = L \times W$
$= 7 \times 14$
$= 98m^2$

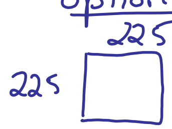
∴ The dimensions are 7m by 14m.

Enclosing non-rectangular areas:

EXAMPLE 5

A farmer is creating a fenced exercise yard for her horses. She has 900m of flexible fencing and wishes to maximize the area. She is going to fence a rectangular or circular area. Determine which encloses the greatest area.

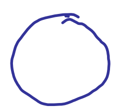
Option 1



$P = 900m$
$L = \frac{900m}{4}$
$= 225m$

$Area = L \times W$
$= 225 \times 225$
$= 50625 m^2$

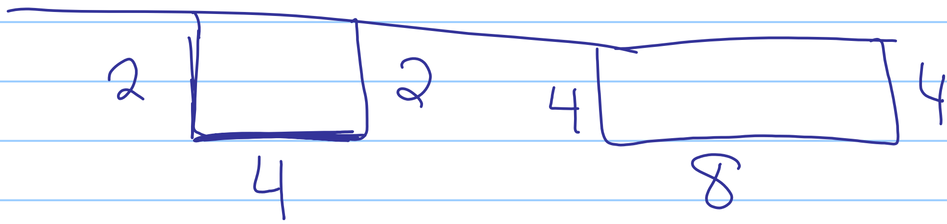
Option 2



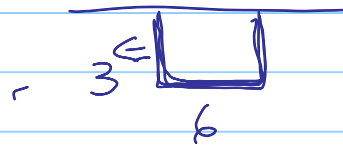
$C = 900m$
$C = 2\pi r$
$\frac{900}{2\pi} = \frac{2\pi r}{2\pi}$
$143.24 = r$

$A = \pi r^2$
$= \pi (143.24)^2$
$= 64457.75 m^2$.

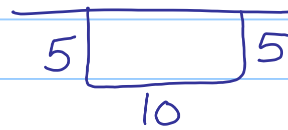
∴ The circle encloses the largest area.



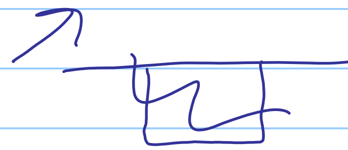
Area = $18m^2$.



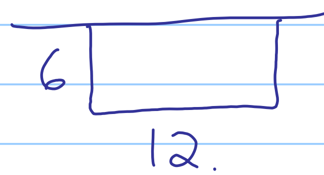
Perimeter = 20m.



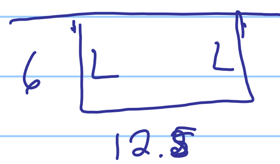
Area = $50m^2$



Area = $72m^2$



Area = $75m^2$



$A = L \times 2L$

$$\frac{75}{2} = \frac{2L^2}{2}$$

$$\sqrt{37.5} = \sqrt{L^2}$$

$6.1 \text{ m}^2 = L$

$W = 12.2 \text{ m}^2$