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## Lesson 1.3 - Obtuse Triangle Investigation

- Learning Goals: Investigate connections between primary trigonometric ratios of acute angles and obtuse angles. Determine the values of the sine ratio, cosine ratio, and tangent ratio for obtuse angles.


## Recall

Angles can be divided into three groups based on their measure:


Today we will look at sin, cos and tan ratios for obtuse angles. Can we draw a right triangle with one obtuse angle? Explain. No. Since angles in a triangle add to $150^{\circ}$,
if one angle is $90^{\circ}$ other two add to $90^{\circ}$ Instead, we use a Cartesian coordinate system to think about trigonometric ratio:

QI: $0<\theta<90$
${ }_{4} 02: 90<\theta<180$
QB: ${ }^{180}<\theta<270$

Qu: $270<360$


Two angles are supplementary if they add to $180^{\circ}$

Complete the investigation at http://msrouhani.wikispaces.com/file/view/3025\ 0btuse\ Angles.swf
Complete the worksheet as you answer the questions along the way:
Page 2: $\quad$ Copy down the answers in the table (after you move the point $D$ ).

| $m \angle A B D$ | $m \angle C B D$ | $m \angle A B D+m \angle C B D$ | $\sin (m \angle A B D)$ | $\sin (m \angle C B D)$ | $\cos (m \angle A B D)$ | $\cos (m \angle C B D)$ | $\tan (m \angle A B D)$ | $\tan (m \angle C B D)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

- What do you notice about the values of $\sin (\angle A B D)$ and $\sin (\angle C B D)$ ?
- What about the values $\cos (\angle A B D)$ and $\cos (\angle C B D)$ or the value of $\tan (\angle A B D)$ and $\tan (\angle C B D)$ ?
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## Page 3: After completing the activity, copy the answers below

The angles ABD and CBD are supplementary angles, they add to $180^{\circ}$.
Can you determine the trig ratios of any obtuse angle $\theta$ by using supplementary angles?


For example, given that $\sin 30^{\circ}=0.5, \cos 30^{\circ}=0.866, \tan 30^{\circ}=0.577$, and $150^{\circ}=180^{\circ}-30^{\circ}$
or, given that $\sin 65^{\circ}=0.906, \cos 65^{\circ}=0.423$, and $\tan 65^{\circ}=2.145$, and $115^{\circ}=180^{\circ}-65^{\circ}$


In general, for any obtuse angle $\theta$,

$\sin \theta=\sin (180 \cdot \theta) \quad \cos \theta=-\cos (180-\theta) \quad \tan \theta=-\tan (180-\theta)$
Page 4: After completing the activity, copy the answers below

The angles $\left[\begin{array}{c}180.74 \\ =106^{\circ}\end{array}\right]$ and $74^{\circ}$ are examples of supplementary angles.
Given that $\sin 47^{\circ}=0.731, \sin \begin{gathered}180-47 \\ =133\end{gathered}=0.731$.
Given that $\cos 52^{\circ}=0.616, \cos 128^{\circ}=-0.616$.
An obtuse angle is between $90^{\circ}$ and $180^{\circ}$.
The trig ratios cosine and tangent of an obtuse angle is always a negative value.
Given that $\tan 129^{\circ}=-1.235, \tan 51^{\circ}=1.235$
Given that $\cos 131^{\circ}=-0.656, \cos \left[\begin{array}{c}180131 \\ =49^{\circ}\end{array}=0.656\right.$.

## Page 5

What are the coordinates of the point A in this graph? ( $\qquad$ 3 $\qquad$ ) How do the values of $x$ and $y$ relate to the adjacent and opposite sides of the triangle? $x=$ adjacent side

$$
y=\text { opposite side. }
$$


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## Page 6: $\quad$ Express the trig ratios for the angle $\theta$ in terms of $x, y$, and $r$

$$
\begin{aligned}
& \sin \theta=\frac{\text { opposite }}{\text { hypotenuse }}=\frac{y}{\square} \\
& \cos \theta=\frac{\text { adjacent }}{\text { hypotenuse }}=x \\
& \tan \theta=\frac{\text { opposite }}{\text { adjacent }}=y
\end{aligned}
$$



Page 7: $\quad$ Drag point $P$ to form acute angles. Drag the point $P$ to form an obtuse angle.

- What do you notice about the three trig ratios? Why does this occur? ' $x$ ' co-ordinate will be negative. Hence, $\tan \theta$ will be negative $\sin \in \cos \theta=\frac{x}{r}$ and $\tan \theta=\frac{y}{x}$


## Page 8:

## Summary

For any angle $\theta$ in standard position with the point $\mathrm{P}(\mathrm{x}, \mathrm{y})$ on the terminal $\operatorname{arm}, \sin \theta=\frac{\mathrm{y}}{\mathrm{r}}, \cos \theta=\frac{\mathrm{x}}{\mathrm{r}}$, and $\tan \theta=\frac{y}{x}$, where $r=\sqrt{x^{2}+y^{2}}$.

The trig ratios of an obtuse angle are the same as the trig ratios of the related acute angle but with opposite signs for cosine and tangent.

$$
\sin \theta=\sin \left(180^{\circ}-\theta\right) \quad \cos \theta=-\cos \left(180^{\circ}-\theta\right) \quad \tan \theta=-\tan \left(180^{\circ}-\theta\right)
$$

The sine ratio always stays positive for an acute or obtuse angle since $y$ is positive in the first and second quadrants and $r$, the hypotenuse, is always positive.

The cosine and tangent ratios become negative for an obtuse angle (between $90^{\circ}$ and $180^{\circ}$ ) since x is negative in the second quadrant, which in turn, makes the ratio negative.


Practice: Continue to complete the rest of the pages in the online activity.
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## Lesson 1.4 - Trigonometric Ratios for Obtuse Triangles

* Learning Goals: Investigate connections between primary trigonometric ratios of acute angles and obtuse angles. Determine the values of the sine ratio, cosine ratio, and tangent ratio for obtuse angles.

Cartesian coordinate system is divided into 4 quadrants.

- Label the quadrants on the grid
- Identify the opposite, adjacent and hypotenuse of angle $\theta$
- What is the $x$-coordinate of point $P$ ? What is the $y$-coordinate of point P? 4

$$
a^{2}+b^{2}=c^{2}
$$



- Write the 3 trigonometric ratios

$$
\sin 53.1=\frac{4}{5} \quad \cos 53.1=\frac{3}{5} \quad \text { an } 53.1=\frac{4}{3} \quad \begin{gather*}
c^{2}=25 \\
c=5
\end{gather*}
$$

Qu

Notice that the length of the adjacent side is the $x$-coordinate and the length of the opposite side is the $y$ coordinate. We can use this idea to find the trigonometric ratios of obtuse angles.


Where are $\sin , \cos$ and tan positive? This is called the CAST rule:
Notice: For an angle between $0^{\circ}$ and $180^{\circ}$,

- If cos or tan are positive the angle is Q1
- If cos or tan are negative the angle is $\quad$ QL
- If $\sin$ is positive the angle could be _Q or Q2

There are always two angles that could give us the same sin ratio. When
 finding the angle, we must report both possibilities.

