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6.1 Implicit Differentiation

Notes

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Derivative at a point – implicit differentiation.

4. Find the equation of any tangent line for $x^2 + y^2 = 4$ at $x = 1$.

$2x + 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$(1)^2 + y^2 = 4$

$y^2 = 3$

$y = \sqrt{3}, -\sqrt{3}$

when $(1, \sqrt{3})$

$m = -\frac{1}{\sqrt{3}}$

$y = -\frac{1}{\sqrt{3}}(x-1) + \sqrt{3}$

when $(1, -\sqrt{3})$

$m = \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}(x-1) - \sqrt{3}$

2nd Derivative – Implicit Differentiation:

Finding the 2nd derivative implicitly is a little trickier than finding it explicitly. Once you have done a few, you'll see it's just a matter of algebraic substitution.

5. Find $\frac{d^2y}{dx^2}$ for $x^2 + y^2 = 16$

$2x + 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y} = -x(y)$

$\frac{d^2y}{dx^2} = \frac{(-1)y - (-x)y'}{y^2} = \frac{-y + x\left(-\frac{x}{y}\right)}{y^2}$

$= \frac{\frac{-y^2}{y} - \frac{x^2}{y}}{y^2} = \frac{-y^2 - x^2}{y} \cdot \frac{1}{y^2}$

$= \frac{-y^2 - x^2}{y^3}$

$= \frac{-(x^2 + y^2)}{y^3} = \frac{-16}{y^3}$

Now summarize what you learned!

$(y)^3$ $y = 3x$ $y' = 3$ $(y^2) \to 2y'y'$ $y^3 \to 3y^2 \cdot y'$

6.1 Implicit Differentiation

Calculus $y = f(x)$ Name: _____ **Practice**

Find $\frac{dy}{dx}$.

1. $4 = 5x^2 + 2y^3$

$0 = 10x + 6y^2 \cdot y'$

$6y^2y' = -10x$

$y' = \frac{-10x}{6y^2} = \frac{-5x}{3y^2}$

2. $5y^2 + 3 = x^2$

$10yy' = 2x$

$y' = \frac{2x}{10y} = \frac{x}{5y}$

3. $3x = y^3 + 4$

$3 = 3y^2 \cdot y'$

$y' = \frac{3}{3y^2} = \frac{1}{y^2}$

4. $x^2 = 4y^3 + 5y^2$

$2x = 12y^2y' + 10yy'$

$y'(12y^2 + 10y) = 2x$

$y' = \frac{2x}{12y^2 + 10y}$

5. $(4y^3 + 4)^2 = 3x^2$

$2(4y^3+4)(12y^2y') = 6x$

$y' = \frac{6x}{24y^2(4y^3+4)}$

$= \frac{x}{16y^2(y^3+1)}$

6. $2x^3 = (3y^3 + 4)^2$

$6x^2 = 2(3y^3+4)(9y^2 \cdot y')$

$y' = \frac{6x^2}{18y^2(3y^3+4)}$

$= \frac{x^2}{3(y^2)(3y^3+4)}$

7. $-3y + y^3 = 5x$

$-3y' + 3y^2y' = 5$

$y'(3y^2 - 3) = 5$

$y' = \frac{5}{3(y^2-1)}$

8. $5x^3 - 2y = 5y^3$

$15x^2 - 2y' = 15y^2y'$

$15x^2 = 15y^2y' + 2y'$

$15x^2 = y'(15y^2 + 2)$

$y' = \frac{15x^2}{15y^2 + 2}$

9. $(x+y)^2 = 2x$

$2(x+y)(1+y') = 2$

$1 + y' = \frac{2}{2(x+y)}$

$y' = \frac{1}{x+y} - 1$

$y' = \frac{1-x-y}{x+y}$

10. $4x + 1 = y^2$

$4 + 0 = 2yy'$

$y' = \frac{4}{2y}$

$y' = \frac{2}{y}$

11. $3x^2 - 6y^2 + 5 = 9y - 3x$

$6x - 12yy' = 9y' - 3$

$6x + 3 = 9y' + 12yy'$

$y'(9 + 12y) = 6x + 3$

$y' = \frac{6x+3}{9+12y}$

$= \frac{2x+1}{4y+3}$

12. $y^2 - 7y + x^2 - 4x = 10$

$2yy' - 7y' + 2x - 4 = 0$

$y'(2y - 7) = -2x + 4$

$y' = \frac{-2x+4}{2y-7}$