Review For the Final Exam Name:_____

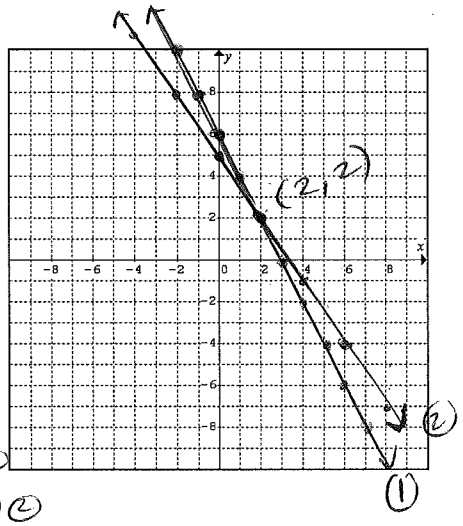
As part of your review, I would suggest that you:

- Write all the Formulas and Definitions used in the course
- Outline the steps needed to complete certain questions
- Complete the suggested review questions
- Redo knowledge and application sections of all tests, quizzes and assignments
- READ your lessons thoroughly

Unit 1 Review – Linear Systems

Topics: Checking point on a line
Solving linear systems (graphing, substitution, elimination)
Intersecting/parallel/coincident systems word problems (numbers, money, investments, mixture, speed/distance/time, currents/headwinds/tailwinds)

1. Solve by graphing: $y=-2x+6$ ① $y=-\frac{3}{2}x+5$ ②



① $m=\frac{-2}{1}$ $b=6$ ∴ POI is (2, 2)

② $m=-\frac{3}{2}$ $b=5$

2. Solve using substitution, and check your answer: $2x+y=6$ ① $3x+2y=10$ ②

From ① $y=-2x+6$ sub into ②

$3x+2(-2x+6)=10$

$3x-4x+12=10$

$-x=-2$

$x=2$ sub in $y=-2x+6$

∴ $y=-2(2)+6=-4+6=2$

∴ The POI is (2, 2)

① LS	RS
$2x+y$	6
$2(2)+2$	
$4+2$	
6	

LS = RS

② LS	RS
$3x+2y$	10
$3(2)+2(2)$	
$6+4$	
10	

LS = RS

∴ POI is (2, 2)

3. Solve using elimination, and check your answer: $4x-3y=25$ ① $-3x+8y=10$ ②

① × 3 | $12x-9y=75$

② × 4 | $-12x+32y=40$

+ | $23y=115$

$y=\frac{115}{23}=5$

Sub $y=5$ in ①

$4x-15=25$

$4x=40$

$x=10$

∴ POI was (10, 5)

① LS	RS
$4x-3y$	25
$4(10)-3(5)$	
$40-15$	
25	

LS = RS

② LS	RS
$-3x+8y$	10
$-3(10)+8(5)$	
$-30+40$	
10	

LS = RS

4. Sue invested $5800, part at 7% and the remainder at 9%. After one year, the total interest earned was $454. How much did she invest at each rate? Let x represent amount invested at 7% and y invested at 9%

$x+y=5800$ ①

$0.07x+0.09y=454$ ②

①×0.07	$0.07x+0.07y=406$
②	$0.07x+0.09y=454$
−	$-0.02y=-48 \Rightarrow y=2400$

$x=5800-2400=3400$

∴ $3400 invested at 7% and $2400 invested at 9%

5. Nick was counting quarters and dimes. He had 124 coins in all, totalling $19.60. How many of each coin did he have? Let q represent number of quarters
Let d represent number of dimes.

$q+d=124$ ①

$0.25q+0.10d=19.60$ ②

From ① $d=124-q$ into ②

$0.25q+0.10(124-q)=19.60$

$0.25q+12.4-0.10q=19.60$

$0.15q=7.20 \Rightarrow q=48$

∴ $d=124-48$
$=76$

∴ Nick had 76 dimes and 48 quarters.

6. Thomas has two part-time jobs delivering flyers. He earns $9/h at his weekend job and $12/h at his weekday job. Last week, he worked 23 hours and earned a total of $231. How many hours did Thomas work at each job? Let x represent hours worked at $9/h (weekend)
Let y represent hours worked at $12/h (weekday)

$x+y=23$ ①

$9x+12y=231$ ②

From ① $y=23-x$ into ②

$9x+12(23-x)=231$

$9x+276-12x=231$

$-3x=231-276$

$-3x=-45$

$x=15$

∴ $y=23-15$
$=8$

∴ Thomas worked for 15 hours in weekend and 8 hours during the week.

7. Coffee that sells for $3.60/kg is blended with coffee that sells for $2.40/kg to make 2 kg of coffee that sells for $2.70/kg. How much of each type of coffee was used?
Let x represent amount of coffee that sells for $3.60/kg.
Let y represent amount of coffee that sells for $2.40/kg.

$x+y=2$ ①

$3.60x+2.40y=2.70(2)$ ②

①×3.6	$3.60x+3.60y=7.20$
②	$3.60x+2.40y=5.40$
−	$1.20y=1.80$

$y=1.5$

$x=0.5$

∴ 0.5 kg of coffee @ $3.60/kg and 1.5 kg of coffee at $2.40/kg.

Suggested review from the textbook: **Page 156 #1ac, 2, 4, 5, 6**

Unit 2 & 3: Analytic Geometry and Geometric Properties

Topics:

- Midpoint of a line segment
- Length of a line segment
- Equations of lines
- Applying slope, midpoint, length
- Equation of a circle
- Properties of triangles
- Properties of quadrilaterals
- Properties of circles

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. The endpoints of a line segment are A(-4, 3) and B(5, -2). Calculate:
   a. The length of AB
   b. The midpoint of AB
   c. The slope of AB

$d = \sqrt{(5+4)^2 + (-2-3)^2}$

$= \sqrt{9^2 + (-5)^2}$

$= \sqrt{81+25} = \sqrt{106}$

$M = \left(\frac{-4+5}{2}, \frac{3+(-2)}{2}\right)$

$= \left(\frac{1}{2}, \frac{1}{2}\right)$

$m = \frac{-2-3}{5-(-4)}$

$= \frac{-5}{9}$

2. Find the equation of the line that passes through the points in question 1.

$y = m(x - x_1) + y_1$ sub $m = \frac{-5}{9}$ and A(-4, 3)

$y = \frac{-5}{9}(x+4) + 3$

$= -\frac{5}{9}x - \frac{20}{9} + 3 = -\frac{5}{9}x - \frac{20}{9} + \frac{27}{9} = -\frac{5}{9}x + \frac{7}{9}$

3. Determine the equation of the following circles:
   a. Centre (0, 0) and radius 4
   b. Centre (0, 0) and through (3, 4)

$x^2 + y^2 = 4^2$

$\therefore x^2 + y^2 = 16$

$x^2 + y^2 = r^2$

$3^2 + 4^2 = r^2$

$9 + 16 = r^2$

$r^2 = 25$

$\therefore x^2 + y^2 = 25$

4. Determine whether the point (1, -9) is inside, outside, or on the circumference of the circle $x^2 + y^2 = 100$.

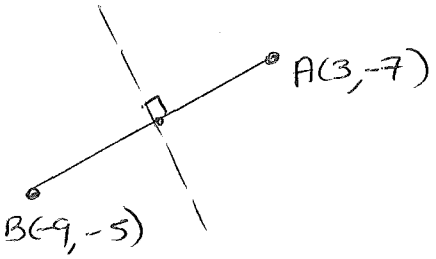
$x^2 + y^2 = 1^2 + (-9)^2$

$= 1 + 81$

$= 82$

Since $82 < 100$, point $(1, -9)$ is inside the circle.

5. Line segment AB has endpoints A(3, -7) and B(-9, -5). Determine the equation of the perpendicular bisector of AB.



$M_{AB} = \left(\frac{3+(-9)}{2}, \frac{-7+(-5)}{2}\right) = \left(\frac{-6}{2}, \frac{-12}{2}\right) = (-3,-6)$

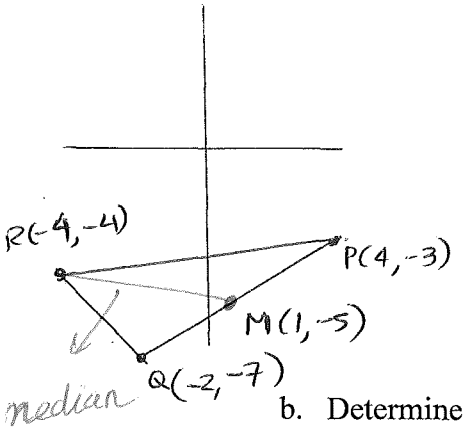
$m_{AB} = \frac{-5+7}{-9-3} = \frac{2}{-12} = -\frac{1}{6}$ $\quad m_{\perp} = 6$

$\therefore y = m(x-x_1)+y_1$

$= 6(x+3)-6 \quad = 6x+18-6$

$\boxed{y = 6x+12}$

6. ΔPQR has vertices P(4, -3), Q(-2, -7) and R(-4, -4).
   a. Classify the triangle as scalene, isosceles, or equilateral.



$d_{PQ} = \sqrt{(4+2)^2+(-3+7)^2} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$d_{PR} = \sqrt{(4+4)^2+(-3+4)^2} = \sqrt{8^2+(1)^2} = \sqrt{65}$

$d_{QR} = \sqrt{(-2+4)^2+(-7+4)^2} = \sqrt{2^2+(-3)^2} = \sqrt{13}$

$\therefore \Delta PQR$ is scalene.

   b. Determine the equation of the median from vertex R?

$M_{PQ} = \left(\frac{4+(-2)}{2}, \frac{-3+(-7)}{2}\right) = \left(\frac{2}{2}, -\frac{10}{2}\right) = (1,-5)$

$m_{MR} = \frac{-5+4}{1+4} = \frac{-1}{5}$

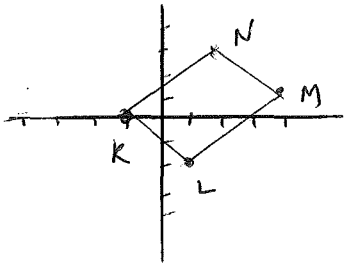
$y = m(x-x_1)+y_1$

$= -\frac{1}{5}(x-1)-5$

$= -\frac{1}{5}x+\frac{1}{5}-5 \quad = -\frac{1}{5}x+\frac{1}{5}-\frac{25}{5} = -\frac{1}{5}x-\frac{24}{5}$

$\therefore y = -\frac{1}{5}x-\frac{24}{5}$ is the eqn of the median

7. Verify that the quadrilateral with vertices K(-1, 0), L(1, -2), M(4, 1) and N(2, 3) is a rectangle.



$d_{KL} = \sqrt{(1+1)^2+(-2-0)^2} = \sqrt{8}$ $\quad m_{KL} = \frac{-2}{2} = -1$

$d_{MN} = \sqrt{(2-4)^2+(3-1)^2} = \sqrt{8}$ $\quad m_{MN} = \frac{2}{-2} = -1$

$d_{LM} = \sqrt{(4-1)^2+(1+2)^2} = \sqrt{18}$ $\quad m_{LM} = \frac{3}{3} = 1$

$d_{KN} = \sqrt{(2+1)^2+(3-0)^2} = \sqrt{18}$ $\quad m_{KN} = \frac{3}{3} = 1$

($m=-1$ is $\perp$ to $m=1$)

$\therefore$ Opposite sides are equal and parallel and adjacent sides are perpendicular

$\therefore$ KLMN is rectangle

Suggested review from the textbook: Page 156# 7, 9, 11ac, 15 & Page 439 # 12

Unit 5 – Quadratic Expressions

Topics:

- Multiplying polynomials
- Special products
- Common factors
- Factoring trinomials
- Factoring difference of squares
- Factoring perfect square trinomials

1. Expand and simplify each:

a. $(2x+5)(3x-2)$

$= 6x^2 - 4x + 15x - 10$

$= 6x^2 + 11x - 10$

b. $3(x+3)(x+2)-(x-2)^2$

$= 3[(x+3)(x+2)] - [(x-2)(x-2)]$

$= 3[x^2 + 2x + 3x + 6] - [x^2 - 4x + 4]$

$= 3x^2 + 6x + 9x + 18 - x^2 + 4x - 4$

$= 2x^2 + 19x + 14$

2. Factor completely:

a. $-4x^2 - 28x$ [GCF]

$= -4x(x+7)$

b. $x^2 - 7x + 12$ [simple]

$= (x-4)(x-3)$

c. $4x^2 - 28x + 40$ [TRICKY]

$= 4(x^2 - 7x + 10)$

$= 4(x-5)(x-2)$

d. $36x^2 - 49$ [DOS]

$= (6x-7)(6x+7)$

e. $10x^2 + x - 3$ [Complex]

M	A	N
-30	1	6, -5

$= 10x^2 + 6x - 5x - 3$

$= 2x(5x+3) - (5x+3)$

$= (2x-1)(5x+3)$

f. $3x^2 - 2x - 8$ [Complex]

M	A	N
-24	-2	-6, 4

$= 3x^2 - 6x + 4x - 8$

$= 3x(x-2) + 4(x-2)$

$= (3x+4)(x-2)$

g. $4x^2 - 11x + 6$ [complex]

M	A	N
24	-11	-8, -3

$= 4x^2 - 8x - 3x + 6$

$= 4x(x-2) - 3(x-2)$

$= (4x-3)(x-2)$

h. $25x^2 - 20x + 4$

$= (5x-2)^2$ ← perfect square trinomial

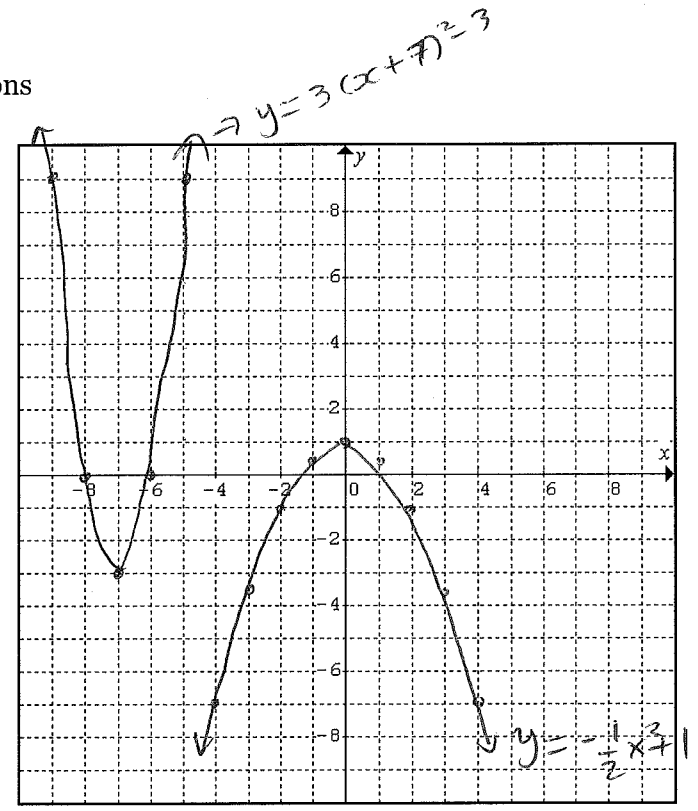
Suggested review: **Page 320# 8, 9, 11, 12**

Unit 4 &6: -Quadratic Relations & Quadratic Equations

Topics:

- Transformations of quadratics
- Graph $y = a(x-h)^2 + k$
- Quadratic relations in the form $y = a(x-r)(x-s)$
- Maxima and Minima
- Solving quadratic equations
- Solving quadratics using x-intercepts
- The quadratic formula
- Solve problems using the quadratic equations

1. For each parabola, state the following and graph:



	$y = -\frac{1}{2}x^2 + 1$	$y = 3(x+7)^2 - 3$
a. direction of opening	down	up
b. the vertex	(0, 1)	(-7, -3)
c. step pattern	$-\frac{1}{2}(1,3,5,7) = -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, -\frac{7}{2}$	$3(1,3,5,7) = 3, 9, 15, 21$
d. the axis of symmetry	$x = 0$	$x = -7$
e. the max or min value	max of 1	min of -3
f. the transformations	• vertically compressed by a factor of $+\frac{1}{2}$ • Reflected on x-axis • vertically shifted 1 unit up	• vertically stretched by a factor of 3 • horizontal shift 7 units left and vertical shift 3 units down.

2. Determine if each forms a linear or quadratic relationship or neither. (find first and second diff)

a.

x	y		
0	7		
1	9	2	
2	15	6	4
3	25	10	4

This is a quadratic relation since the second differences are equal.

b.

x	y	
2	-5	
3	-11	-6
4	-17	-6
5	-23	-6

This is a linear relation since the first differences are constant.

3. Determine the equation of the parabola with vertex at (-8, 1) passing through the point (-1, -48).

$y = a(x-h)^2 + k$

$y = a(x+8)^2 + 1$ sub $x=-1$ $y=-48$ to find 'a'

$-48 = a(-1+8)^2 + 1$

$-48 = 49a + 1$

$-49 = 49a$

$a = -1$

$\therefore y = -(x+8)^2 + 1$

4. Solve by factoring:

a. $x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0$ or $x = -4$

$\{0, -4\}$

b. $x^2 - 36 = 0$

$(x-6)(x+6) = 0$

$x = \pm 6$

$\{\pm 6\}$

c. $x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6$ or $x = 2$

$\{-6, 2\}$

d. $12x^2 + 17x = 5$

$12x^2 + 17x - 5 = 0$

$12x^2 + 20x - 3x - 5 = 0$

M	A	N
-60	17	+20, -3

$4x(3x+5) - 1(3x+5) = 0$

$(4x-1)(3x+5) = 0$

$x = +\frac{1}{4}$ or $-\frac{5}{3}$ $\quad \{+\frac{1}{4}, -\frac{5}{3}\}$

5. Solve using the quadratic formula. Round to 2 decimal places if necessary.

$x^2-4x-1=0$ $a=1, b=-4, c=-1$ $x=\frac{4\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)}$ $=\frac{4\pm\sqrt{20}}{2}=\frac{4\pm4.47}{2}$ $x_1=\frac{4+4.47}{2}\doteq 4.24$ $x_2=\frac{4-4.47}{2}\doteq -0.24$	$-5x^2+8x-10=0$ $x=\frac{-8\pm\sqrt{8^2-4(-5)(-10)}}{2(-5)}$ $=\frac{-8\pm\sqrt{64-200}}{-10}$ $=\frac{-8\pm\sqrt{-136}}{-10}$ No real solution (since discriminant is less than 0)	$4x^2-4x+1=0$ $x=\frac{4\pm\sqrt{(-4)^2-4(4)(1)}}{2(4)}$ $=\frac{4\pm\sqrt{16-16}}{8}$ $=\frac{4\pm0}{8}=\frac{4}{8}=\frac{1}{2}$ (one solution) order 2

6. Determine the vertex by completing the square.

a. $y=3x^2-6x-8$

$=3(x^2-2x)-8$

$=3(x^2-2x+1-1)-8$

$=3(x^2-2x+1)-8-3$

$=3(x-1)^2-11$

b. $y=-x^2-6x+1$

$=-(x^2+6x)+1$

$=-(x^2+6x+9-9)+1$

$=-(x^2+6x+9)+1+9$

$=-(x+3)^2+10$

7. Determine the vertex by factoring/averaging the zeros.

a. $y=-2x^2+14x+36$

$=-2(x^2-7x-18)$

$=-2(x-9)(x+2)$

zeros: $9, -2$

$x_V=\frac{9+(-2)}{2}=3.5$ $y_V=60.5$

$\therefore V(3.5, 60.5)$

b. $y=-4x^2+20x$

$=-4x(x-5)$

zeros: 0 or 5

$x_V=\frac{0+5}{2}=2.5$

$y_V=25$ → sub $x=2.5$ in y.

$\therefore V(2.5, 25)$

Applications of Quadratic Relations

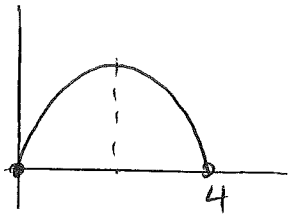
8. The height of a golf ball after being hit is described by $h = -5t^2 + 20t$, where h metres is the height, t seconds after the ball has been hit.

a. Is this ball being hit from the ground or on a tee? How do you know?

On the ground (initial $h=0$)

b. What is the maximum height of the ball? complete the square or average the zeros.

$h = -5t(t-4)$



zeros: 0 or 4

$t_v = \frac{0+4}{2} = 2$

$h_v = -5(2)^2 + 20(2) = 20$

∴ Maximum height was 20 m.

c. How many seconds after impact is the maximum height reached?

2 seconds

d. When does the ball hit the ground?

set $h=0$ solve for t (factor or use quadratic formula)

$-5t^2 + 20t = 0$

$-5t(t-4) = 0$

$t = 0$ or 4

∴ After 4 seconds, the golf ball hits the ground.

e. For how long is the ball over a height of 10 m? Round to one decimal place.

$-5t^2 + 20t = 10$

$5t^2 - 20t + 10 = 0$

$t^2 - 4t + 2 = 0$

Does not factor (use QF)

$t = \frac{4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$

$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2.83}{2}$

$t_1 = \frac{4+2.83}{2} = 3.42$ } subtract

$t_2 = \frac{4-2.83}{2} = 0.59$ }

∴ The ball was over a height of 10 m for 2.83 sec.

Suggested review: **Page 320 #2, 3bc, 15, 16, 17ac, 19ad, 20, 21**

9. The revenue, R, from concert ticket sales at a local venue is calculated as $(number\ of\ tickets\ sold) \times (price\ of\ ticket)$. The current price of a ticket is \$20, and the venue typically sells 100 tickets. For each \$1 decrease in ticket price, 10 more tickets are sold.

a. Write an equation to model the Revenue.

Let x represent number of times price decreases.

$R = (20-x)(100+10x)$

b. What is selling price that will maximize the Revenue? Determine the maximum Revenue.

$R = (20-x)(100+10x)$ Expand and complete the square OR find zeros then find $x_v =$ $y_v =$

Zeros: $x = 20$ or $x = -10$

$x_v = \frac{20+(-10)}{2} = 5$

$R_v = (20-5)(100+10(5)) = (15)(150) = \$2250$

OR $R = 2000 + 200x - 100x - 10x^2$

$= -10x^2 + 100x + 2000$

$= -10(x^2 - 10x) + 2000$

$= -10(x^2 - 10x + 25 - 25) + 2000$

$= -10(x-5)^2 + 2250$

$\therefore$ max $R = \$2250$

Price $= 20 - 5 = \$15$

c. Find the selling price of each ticket if the Revenue was \$2,090.

$R = 2090$

$-10x^2 + 100x + 2000 = 2090$

$-10x^2 + 100x - 90 = 0$

$-10(x^2 - 10x + 9) = 0$

$-10(x-1)(x-9) = 0$

$x = 1$ or $x = 9$

Price $= 20 - 1 = 19$ Price $= 20 - 9 = 11$

$\therefore$ selling price should be \$11 or \$19 for revenue to be \$2090.

Suggested review from the textbook: Page 320 #2, 3bc, 15, 16, 17ac, 19ad, 20, 21

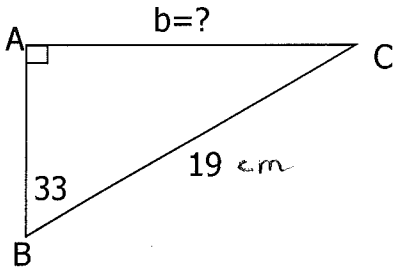
Unit 7/8: Trigonometry

Topics:

- Similar triangles
- The Tangent, Sine and cosine ratios
- Solving problems using two right triangles
- The sine law
- The cosine law

1. Determine the length of the indicated side to the nearest tenth and the angle to the nearest degree.

a.

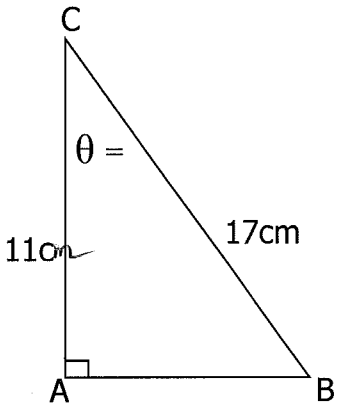


$\sin 33 = \frac{b}{19}$

$b = 19(\sin 33)$

$\doteq 10.4$ cm

b.

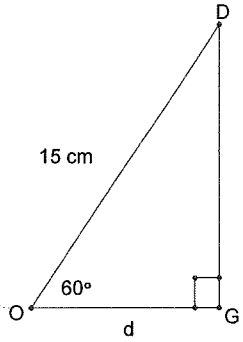


$\cos\theta = \frac{11}{17}$

$\angle\theta = \cos^{-1}\left(\frac{11}{17}\right)$

$\doteq 50°$

c.

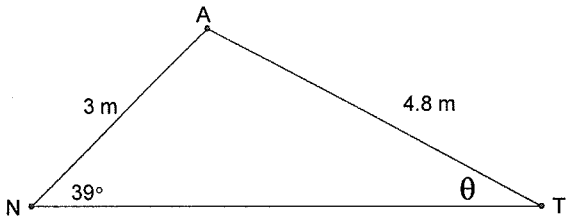


$\cos 60 = \frac{d}{15}$

$d = 15\cos 60$

$= 7.5$ cm

d.



$\frac{\sin\theta}{3} = \frac{\sin 39}{4.8}$

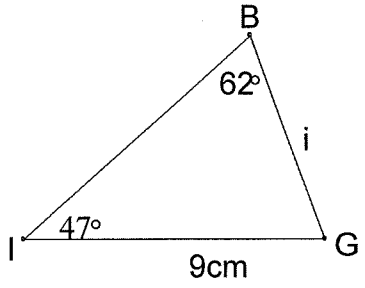
$\sin\theta = \frac{3(\sin 39)}{4.8}$

$\sin\theta = 0.393$

$\angle\theta = \sin^{-1}(0.393)$

$\doteq 23°$

e.

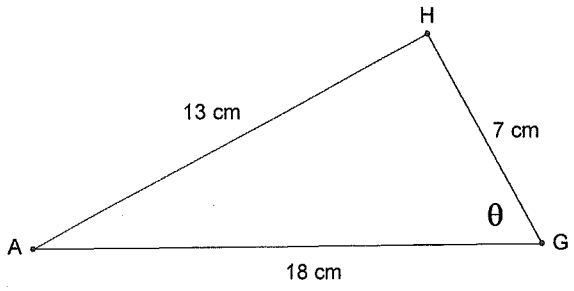


$\frac{i}{\sin 47} = \frac{9}{\sin 62}$

$i = \frac{9(\sin 47)}{\sin 62}$

$\doteq 7.5$ cm

f.



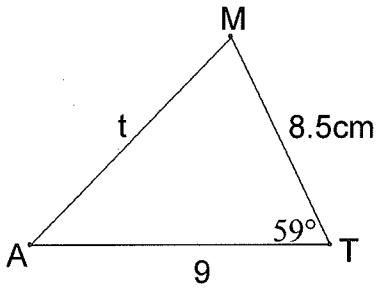
$\cos\theta = \dfrac{7^2+18^2-13^2}{2(7)(18)}$

$\cos\theta = 0.8095$

$\angle\theta = \cos^{-1}(0.8095)$

$\doteq 36^\circ$

g.



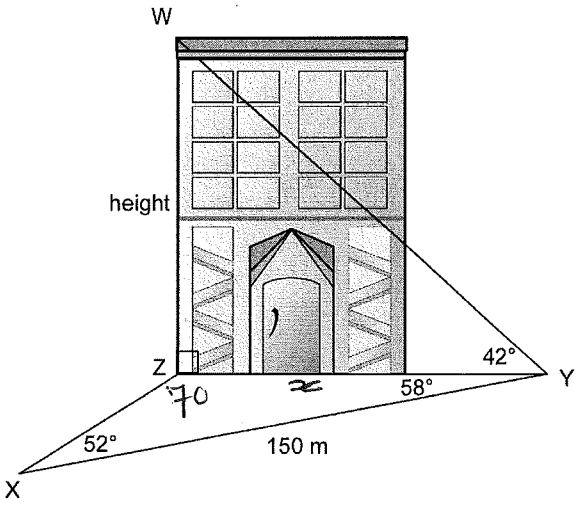
$t^2 = 9^2+8.5^2-2(9)(8.5)\cos 59^\circ$

$t^2 = 74.45$

$t = \sqrt{74.45}$

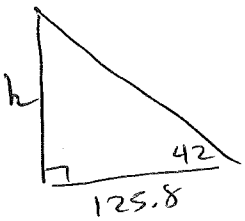
$\doteq 8.6\text{ cm}$

2. Use the measurements given in the diagram to find the height of the building.



$\dfrac{x}{\sin 52^\circ} = \dfrac{150}{\sin 70}$

$x = \dfrac{150(\sin 52)}{\sin 70} \doteq 125.8\text{ m}$



$\tan 42 = \dfrac{h}{125.8}$

$h = 125.8(\tan 42)$

$\doteq 113.3$

∴ The height of the building was 113.3 m

Suggested review from the textbook: Page 434#2, 3a, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15

FINAL COURSE REVIEW

Page 438# 3a, 4d, 5, 9, 11, 14, 19b, 20, 24, 25a, 36c, 37, 42acdf, 44acf, 45bef, 46, 48abc, 49ade, 53b, 55bd, 56abc, 60ac, 66, 68, 69, 75, 77, 80