Name: SOLUTIONS MAP4C1 Date: _____

Geometry Review

You will be given a reference sheet with conversions and formulas for area, perimeter, volume and surface area.

Communication in all questions must include:
- Enough steps shown to clearly demonstrate thinking
- Solutions that are neat and easy to follow
- Proper use of mathematical symbols
- Equal signs aligned
- Units used as required
- Concluding statements for all word problems
- Fractions reduced to lowest terms
- Correct rounding.

Conversions:

1. Convert each measurement as indicated:
   a. 4.2 metres to feet and inches
   b. 62 centimeters to metres and inches
   c. 16 feet to metres and inches
   d. Convert 4 litres to quarts and pints
   e. Convert 10 pints to litres and quarts

a) $4.2\text{m} \times \frac{3.2808\text{ft}}{1\text{m}} = 13.77936\text{ft}$

$4.2\text{m} \times \frac{39.37\text{ inches}}{\text{m}} = 165.3876$

b) $62\text{cm} \times \frac{100\text{m}}{\text{cm}} = 6200\text{m}$

$62\text{cm} \times \frac{0.3937\text{in}}{1\text{cm}} = 24.4094\text{in}$

c) $16\text{ft} \times \frac{0.3048\text{m}}{\text{ft}} = 4.8768\text{m}$

$4.8768\text{m} \times \frac{39.37\text{in}}{\text{m}} = 191.999616\text{in}$

d) $4\text{L} \times \frac{0.8799\text{quarts}}{\text{L}} = 3.5196\text{quarts}$

$4\text{L} \times \frac{1.7598\text{pints}}{\text{L}} = 7.0392\text{pints}$

e) $10\text{pints} \times \frac{0.568\text{L}}{1\text{pt}} = 5.68\text{L}$

$5.68\text{L} \times \frac{0.8799\text{quarts}}{\text{L}} = 4.997832\text{ quarts}$

2. Which is more, 90 metres or 300 feet?

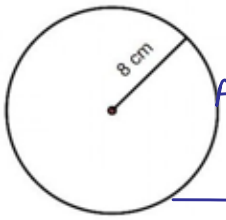
$90\text{m} \times \frac{3.2808\text{ft}}{\text{m}} = 295.272\text{ft}$

$\therefore$ 300ft is more

2D Geometry

3. Find the area and perimeter of each shape:

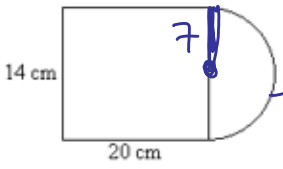
a)



$A = \pi r^2 = \pi(8)^2 = 201.06\text{cm}^2$

$C = 2\pi r = 2\pi(8) = 50.27\text{cm}$

b)



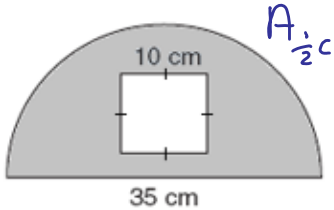
$A_{square} = 14 \times 20 = 280\text{cm}^2$

$A_{\frac{1}{2}circle} = \frac{1}{2}(\pi r^2) = \frac{1}{2}\pi(7)^2 = 76.97\text{cm}^2$

$A_{Total} = 280\text{cm}^2 + 76.97\text{cm}^2 = 356.97\text{cm}^2$

$P = 14 + 20 + 20 + \frac{2\pi(7)}{2} = 54 + 21.99 = 1187.52\text{cm}$

c)



$r = \frac{35}{2} = 17.5$

$A_{\frac{1}{2}circle} = \frac{1}{2}(\pi r^2) = \frac{1}{2}\pi(17.5)^2 = 481.06\text{cm}^2$

$A_{square} = L \times W = 10 \times 10 = 100\text{cm}^2$

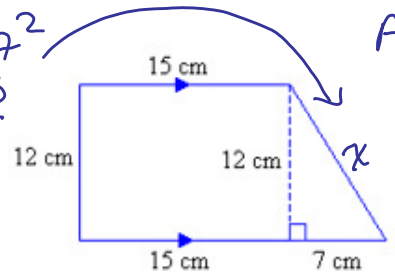
$A_{Total} = 481.06 - 100 = 381.06\text{cm}^2$

$P_{half\ circle} = \frac{1}{2}(2\pi r) + d = \frac{1}{2}(2\pi(17.5)) + 35 = 89.98\text{cm}$

$P_{Total} = 89.98 + 4(10) = 129.98\text{cm}$

$x^2 = 12^2 + 7^2$
$x^2 = 193$
$x = \sqrt{193}$
$= 13.89$ cm

d)



$A_{square} = 15 \times 12$
$= 180 cm^2$

$A_{Triangle} = \frac{12 \times 7}{2}$
$= 42 cm^2$

$A_{total} = 180 cm^2 + 42 cm^2$
$= 222 cm^2$

$P = 12 + 15 + 15 + 7 + 13.89$
$= 62.89$ cm.

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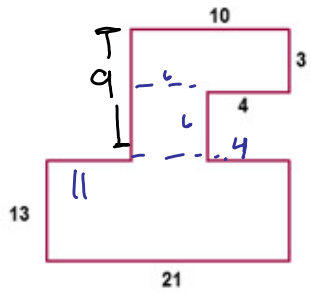
e)



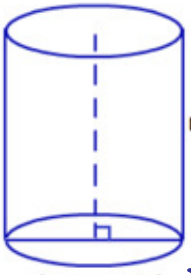
$A = 10 \times 3 + 6 \times 6 + 13 \times 21$
$= 30 + 36 + 273$
$= 339 cm^2$

$P = 9 + 10 + 3 + 4 + 6 + 4 + 13 + 21 + 13 + 11$
$= 94$ cm.

3D Geometry

4. Find the surface area and volume of each figure

a)



Diameter = 10 ft $\rightarrow r = \frac{10}{2} = 5$ ft.

$SA = 2\pi rh + 2\pi r^2$
$= 2\pi(5)(21) + 2\pi(5)^2$
$= 659.73 + 157.08$
$= 816.81 ft^2$

$V = \pi r^2 h$
$= \pi(5)^2(21)$
$= 1649.34 ft^3$

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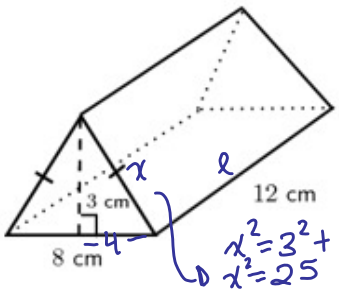
b)



$x^2 = 3^2 + 4^2$
$x^2 = 25$
$x = 5$

$SA = 2\left(\frac{bh}{2}\right) + 2(lx) + bl$
$= 2\left(\frac{8 \times 3}{2}\right) + 2(12 \times 5) + (8 \times 12)$
$= 24 + 120 + 96$
$= 240 cm^2$

$V = \frac{lbh}{2}$
$= \frac{12 \times 8 \times 3}{2}$
$= 144 cm^3$

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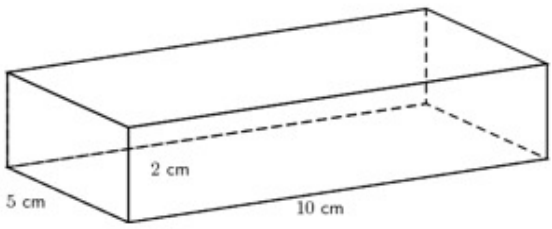
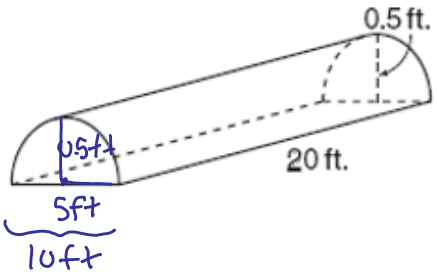
c)



$SA = 2(lw) + 2(lh) + 2(wh)$
$= 2(10 \times 5) + 2(10 \times 2) + 2(5 \times 2)$
$= 100 + 40 + 20$
$= 160 cm^2$

$V = lwh$
$= 10 \times 5 \times 2$
$= 100 cm^3$

d)



Curved top + front and back + bottom

$SA = \frac{1}{2}(2\pi rh) + \frac{1}{2}(\pi r^2)\times 2 + lw$

$= \frac{1}{2}(2\pi(0.5)(20)) + \frac{1}{2}\pi(0.5)^2\times 2 + (20\times 10)$

$= 10\pi + 0.25\pi + 200$

$= 232.2\,ft^2$

$V = \frac{\pi r^2 h}{2}$

$= \frac{\pi(0.5)^2(20)}{2}$

$= 2.5\pi$

$= 7.85\,ft^3$

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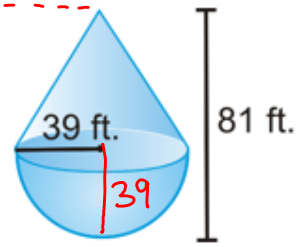
e)



81-39 = 42

*Slant = 50 ft. long

Cone + Sphere

$SA = \pi rs + \frac{1}{2}(4\pi r^2)$

$= \pi(39)(50) + \frac{1}{2}(4\pi(39)^2)$

$= 6126.11 + 9556.72$

$= 15682.83\,ft^2$

$V = \frac{\pi r^2 h}{3} + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

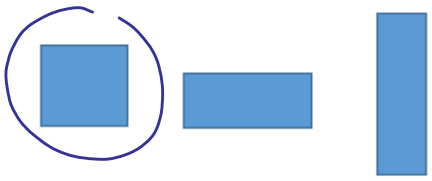
$= \frac{\pi(39)^2(42)}{3} + \frac{1}{2}\left(\frac{4}{3}\pi(39)^3\right)$

$= 66897.07 + 124237.42$

$= 191134.49\,ft^3$

Optimization

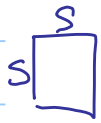
5. The three rectangles shown all have the same perimeter. Which has the largest volume. Explain in words how you made your choice.



The first rectangle would have the largest volume. We know this because it is the most "square-like", meaning the length and width are almost equal. A square is the most optimal rectangle.

6. A rectangular room needs to have an area of 60 m^2. What are the dimensions of the room with a minimum perimeter, and what is the perimeter?

fixed area, minimum perimeter ⇒ room must be a square



$A_{square} = s^2$

$60m^2 = s^2$

$\sqrt{60} = s$

$7.75 = s$

$P = 4s$

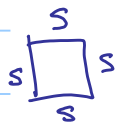
$= 4(7.75)$

$= 30.98m$

∴ The dimensions are 7.75m by 7.75m. The perimeter is 30.98m

7. You need to build a rectangular enclosure in your back-yard. You buy some prebuilt sections of fencing which are each 0.25m long. You buy a total of 40m of fencing. Determine the dimensions (length and width) which will maximize the area of your enclosure

Fixed perimeter, maximize volume ⇒ enclosure must be a square



$P = 4s$

$\frac{40}{4} = \frac{4s}{4}$

$10 = s$

$A = s^2$

$A = 10^2$

$A = 100\,m^2$

∴ The length and width are both 10m. The area is $100m^2$.

8. 35 m of rope are available to create a rectangular swimming area, using the beach as one side. What is the maximum area that can be produced?

Max area using only 3 sides ⇒ Length = 2w

$P = w + w + 2w$

$35 = 4w$

$\frac{35}{4} = w$

$8.75\text{m} = w$

length $= 2w$

$= 2(8.75)$

$= 17.5\text{ m}$

Area $= L \times W$

$= 17.5 \times 8.75$

$= 153.13\text{m}^2$

∴ The maximum area is 153.13m^2

9. A marine biologist is collecting data. She has 100 m of rope with buoys to outline a rectangular or circular research area on the surface of the water. Which figure will enclose a greater area? Justify your answer by showing all calculations.

Rectangular:

(We want the greatest area so it will be a square)

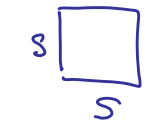
$P = 100\text{m}$

$P = 4s$

$100 = 4s$

$\frac{100}{4} = s$

$s = 25\text{m}$



$\text{Area} = s^2$

$= 25 \times 25$

$= 625\text{m}^2$

Circular

$C = 2\pi r$

$100 = 2\pi r$

$\frac{100}{2\pi} = r$

$15.9\text{m} = r$

$\text{Area} = \pi r^2$

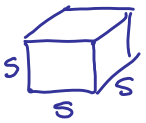
$= \pi(15.9)^2$

$= 794.23\text{m}^2$

∴ The circle would enclose a greater area.

10. A square-based prism must have a surface area of 96 cm^2. What are the dimensions of the prism that produce the maximum volume, and what is the volume?

Maximum Volume ⇒ shape must be a cube



$SA = 6s^2$

$\frac{96}{6} = \frac{6s^2}{6}$

$16 = s^2$

$\sqrt{16} = s$

$4 = s$

$V = s^3$

$= 4^2$

$= 64\text{m}^3$

∴ The dimensions are $4\text{m} \times 4\text{m} \times 4\text{m}$.

The volume is 64m^3

11. A square-based prism has a volume of 50 in^3. Determine the minimum surface area.

Minimum Surface Area ⇒ shape must be a cube.

$V = s^3$

$50\text{in}^3 = s^3$

$\sqrt[3]{50} = s$

$3.68\text{in} = s$

$SA = 6s^2$

$= 6(3.68)^2$

$= 81.43\text{in}^2$

∴ The minimum surface area is 81.43in^2