MPM2D Day 4: Graph Quadratics Using the x-intercepts Date: _____

The vertex in Factored Form

If the zeros of a quadratic relation are r and s, then the x-coordinate (x_v) of the vertex is $\frac{r+s}{2}$. We can find the y-coordinate of the vertex by substituting the x-value of the vertex in the equation given.

Ex 1: What is the vertex of $y = 2(x-2)(x-6)$?

Step 1: What are the zeros?

$x = 2, 6$

Step 2: What is the axis of symmetry OR x_v?

$x_v = \frac{2+6}{2} = 4$

Step 3: What is the optimal value (y_v)?

$y_v = 2(4-2)(4-6)$

$= (2)(2)(-2) = -8$

vertex $(4, -8)$

Ex 2: What is the vertex of $y = (2x+5)(x-6)$?

$x = -\frac{5}{2}, 6$ $\quad [2x+5=0]$

$x = -\frac{5}{2}$

$x_v = \frac{-\frac{5}{2}+6}{2} = \frac{-2.5+6}{2}$

$= \frac{3.5}{2} = 1.75$

$y_v = -36.125$ (sub $x = 1.75$)

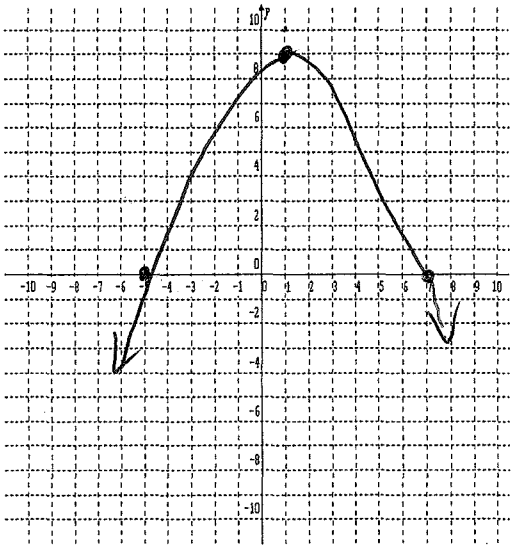
Vertex $(1.75, -36.125)$

Ex 3: Graph $y = -\frac{1}{4}(x+5)(x-7)$ by first finding the vertex?

$x = -5, 7$ (x-intercepts).

$x_v = \frac{-5+7}{2} = 1$

$y_v = \frac{-1}{4}(1+5)(1-7) = \frac{36}{4} = 9$



Ex 4: Find the vertex of $y = \frac{1}{2}(x+2)(x+6)$ then graph the relation.

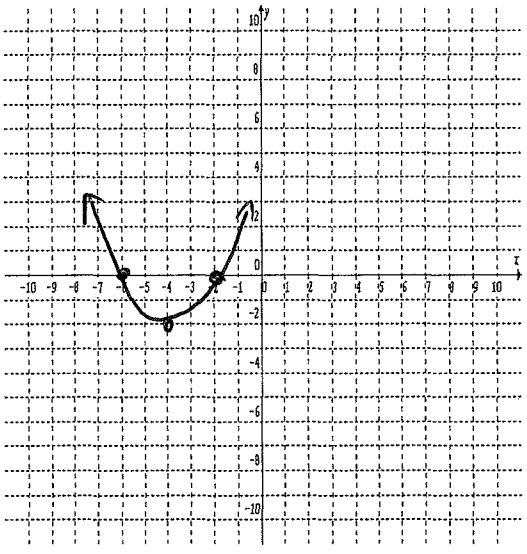
zeros: $-2, -6$

$x_v = \frac{-2+(-6)}{2} = -4$

$y_v = \frac{1}{2}(-4+2)(-4+6)$

$= \frac{1}{2}(-2)(2)$

$= -2$ $\quad V(-4,-2)$



Ex 5: Describe the transformations of quadratic relation with x-intercepts at -1 and 5, and passing through the point $(3,4)$. vertex form (we need)

$y = a(x-r)(x-s)$

$y = a(x+1)(x-5)$

sub $x=3$, $y=4$

$4 = a(3+1)(3-5)$

$4 = a(4)(-2)$

$a = -\frac{1}{2}$

$y = -\frac{1}{2}(x+1)(x-5)$

$x_v = \frac{-1+5}{2} = 2$

$y_v = \frac{-1}{2}(2+1)(2-5) = \frac{9}{2}$ $\quad V(2, \frac{9}{2})$

- Vertically compressed by a factor of $\frac{1}{2}$
- Reflection around x-axis
- Horizontal shift 2 units to the R
- Vertical shift 4.5 units UP.

Extra Practice: **Standard Form, Factored Form, Vertex Form**

1. Fill in the missing entries.

	Standard form	Factored form	x-intercepts	Vertex	Vertex form
(1)	$y = 2x^2 + x - 6$	$y=(2x-3)(x+2)$	$(\frac{3}{2},0)$ $(-2,0)$	$x_v = -0.25$ $y_v = -6.125$	$y=2(x+0.25)^2-6.125$
(2)	$y = x^2 - 7x - 18$	$y=(x-9)(x+2)$	$(9,0)$ $(-2,0)$	$x_v = 3.5$ $y_v = -30.25$	$y=(x-3.5)^2-30.25$
(3)	$y=-3(x^2-2x+1)+27$ $=-3x^2+6x+24$	$y=-3(x^2-2x-8)$ $=-3(x-4)(x+2)$	$(4,0)$ $(-2,0)$	$x_v=1$ $y_v=27$	$y = -3(x-1)^2 + 27$
(4)	$y=x^2-4x-12$	$y=(x+2)(x-6)$	$(-2,0)$ $(6,0)$	$x_v=2$ $y_v=-16$	$y=(x-2)^2-16$
(5)	$y=2x^2+11x-21$	$y=(2x-3)(x+7)$	$(\frac{3}{2},0)$ $(-7,0)$	$x_v=-2.75$ $y_v=-36.125$	$y=2(x+2.75)^2-36.125$
(6)	$y=-\frac{2}{5}(x^2-2x-24)$ $=-\frac{2}{5}x^2+\frac{4}{5}x+\frac{48}{5}$	$y=a(x+4)(x-6)$ Sub $x=1$ $y=10$ $a=-2/5$	$(-4, 0)$ and $(6,0)$	$(1, 10)$	$y=-\frac{2}{5}(x-1)^2+10$ a h k
(7)	$y = x^2 + 6x - 16$	$y=(x-2)(x+8)$	$(2,0)$ $(-8,0)$	$x_v=-3$ $y_v=-25$	$y=(x+3)^2-25$
(8)	$y=x^2-2x-15$	$y=a(x-5)(x+3)$ Sub $x=1$ $y=-16$ $a=1$	$(5, 0)$ and $(-3, 0)$	$(1, -16)$	$y=(x-1)^2-16$
(9)	$y = x^2 - 25$	$y=(x-5)(x+5)$	$(5,0)$ $(-5,0)$	$(0,-25)$	$y=x^2-25$
(10)	$y=x^2-6x+9$	$y=(x-3)^2$	$(3,0)$ order 2	$(3,0)$	$y=(x-3)^2$

2. Express each equation in standard form and vertex form.

   a. $y=(x-4)^2-1$ b. $y=2(x+1)^2-18$

3. The graph of a quadratic relation has zeros at (2,0) and (-6,0) and passes through the point (3,5). Write the equation that models this relation. What is its vertex?
4. The x-intercepts of a parabola are -3 and 5. The parabola crosses the y-axis at -75. Determine the coordinates of the vertex.
5. The graph of $y=-2(x+5)^2+8$ is translated so that its new zeros are -4 and 2. Determine the translation that was applied to the original graph.

Homework: p. 288 #1ac, 2ac, 3bd, 4ac, 7ac, 9, 10

② a) $y = (x-4)^2 - 1$

$= (x-4)(x-4) - 1$

$= x^2 - 8x - 4x + 16 - 1$

$= x^2 - 8x + 15$

b) $y = 2(x+1)^2 - 18$

$= 2(x+1)(x+1) - 18$

$= 2(x^2 + 2x + 1) - 18$

$= 2x^2 + 4x + 2 - 18$

$= 2x^2 + 4x - 16$

③ $y = a(x+6)(x-2)$

sub $x = 3$ $y = 5$

$5 = a(3+6)(3-2)$

$5 = a(9)(1)$

$a = 5/9$

$y = \frac{5}{9}(x+6)(x-2)$

④ x-ints are $-3, 5$ y-int -75

$\therefore y = a(x+3)(x-5)$

sub $x = 0$ $y = -75$

$-75 = a(3)(-5)$

$-75 = -15a$

$a = \frac{-75}{-15}$

$= 5$

$\therefore y = 5(x+3)(x-5)$

(5) $y = -2(x+5)^2 + 8$

New zeros: $-4, 2$

Determine new transformations

$y = a(x+4)(x-2)$ $\quad a = -2$

$= -2(x+4)(x-2)$

$x_v = \frac{-4+2}{2} = -1$

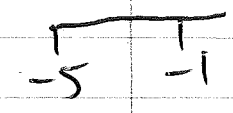
$y_v = -2(-1+4)(-1-2)$

$= -2(3)(-3)$

$= 18$

$V(-1, 18)$ initial $V(-5, 8)$

∴ Transformations are: horizontal shift 4 units to the right



vertical shift 10 units up