MHF4U Exam Review

Unit 1 Polynomial Functions

1. Find the quotient and the remainder of :

a) $\frac{a^3-12a+7}{a+4}$

$-4 \mid 1 \quad 0 \quad -12 \quad 7$
$\quad\quad\downarrow \quad -4 \quad 16 \quad -16$
$\quad\quad 1 \quad -4 \quad 4 \mid -9$
$\quad\quad\quad\quad\quad\quad\quad\quad\downarrow$
$\quad\quad\quad\quad\quad\quad\quad\quad R$

Quotient: x^2-4x+4

Remainder: $-9 \rightarrow P(-4)$

b) $(6x^3+x^2-4x-3) \div (2x+3)$

$-\frac{3}{2} \mid 6 \quad 1 \quad -4 \quad -3$
$\quad\quad\downarrow \quad -9 \quad 12 \quad -12$
$\quad\quad 6 \quad -8 \quad 8 \quad -15$

$Q: 3x^2-4x+4$

$R = -15$

2. Find the remainder of:

a) $(3m^3+7m^2-2m-11) \div (m-2)$

$P(m)$

$P(2) = 3(2)^3 + 7(2)^2 - 2(2) - 11$
$= 24 + 28 - 15$
$= 37$

b) $(4y^3+6y^2-4y-3) \div (2y+1)$

$P(y)$

$P(-1/2) = 0$

3. The zeros of a polynomial f(x) are - 2, $\frac{-1}{2}$, and 3. What is a possible equation for the function f(x)?

$f(x) = a(x+2)(2x+1)(x-3), a \neq 0$

eg: $f(x) = (x+2)(2x+1)(x-3)$

4. Factor completely. $P(1) = 0$

a) $2x^3-7x^2+2x+3$ $x-1$ is a factor

$1 \mid 2 \quad -7 \quad 2 \quad 3$
$\quad\quad\downarrow \quad 2 \quad -5 \quad -3$
$\quad\quad 2 \quad -5 \quad -3 \quad 0$

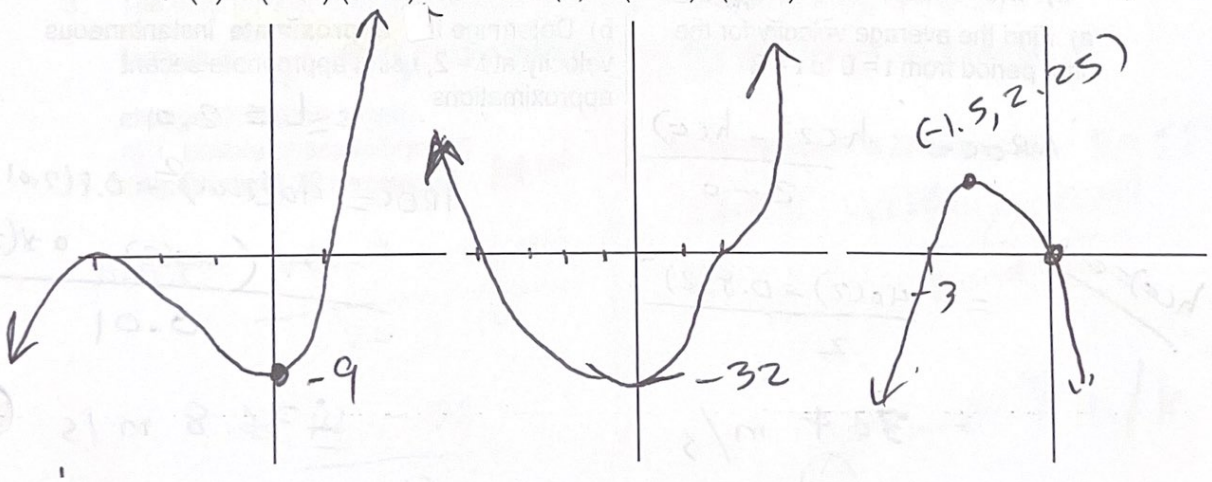
$P(x) = (x-1)(2x+1)(x-3)$

b) $\frac{1}{2}m^3 - 4 = \frac{1}{2}(m^3-8)$

$= \frac{1}{2}(m-2)(m^2+2m+4)$

5. Sketch each of the following graphs on separate grids

a) $f(x) = (x-1)(x+3)^2$ b) $g(x) = (x-2)^3(x+4)$ c) $h(x) = -x(x+3)$



6. Find k so that $x+5$ is a factor of $2x^3+9x^2+kx-15$. $P(-5)=0$

$2(-5)^3+9(-5)^2+k(-5)-15=0$

$-250+225-5k-15=0$

$-5k=40 \Rightarrow k=-8$

7. Find the family of quartic functions whose x-intercepts are - 2, - 1, 1 and 3 ; then find the particular member of the above family whose graph passes through the point (2, - 6)

$y=a(x+2)(x+1)(x-1)(x-3),\ a\neq 0$

Sub $x=2,\ y=-6 \Rightarrow a=\frac{1}{2}$

$y=\frac{1}{2}(x+2)(x+1)(x-1)(x-3)$

8. Find the **exact** value of x, $x\in C$.

a) $x^3-2x^2-5x+6=0$ $P(1)=0$

1	1	−2	−5	+6
		1	−1	−6
	1	−1	−6	0

$x=\{-2,1,3\}$

$(x-3)(x+2)(x-1)=0$

b) $x^2(x+1)=5x+2$

$x^3+x^2-5x-2=0$

$P(2)=0$

2	1	1	−5	−2
		2	6	2
	1	3	1	0

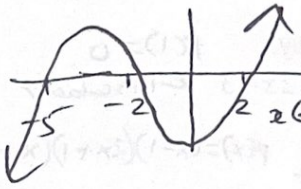
$(x-2)(x^2+3x+1)=0$

$x=2 \quad x=\frac{-3\pm\sqrt{5}}{2}$

9. Sketch a graph to solve the inequality $x^3+5x^2-4x-20<0$.

$x^2(x+5)-4(x+5)<0$

$(x-2)(x+2)(x+5)<0$



$x\in(-\infty,-5)\cup(-2,2)$

10. If a projectile is fired straight upward from the surface of the moon with an initial upward velocity of 40 m/s, its height in metres after t seconds is given by: $h(t)=40t-0.8t^2$. AROC

a) Find the average velocity for the time period from t = 0 to t = 2

$AROC=\frac{h(2)-h(0)}{2-0}$

$h(0)=0$

$=\frac{40(2)-0.8(2)^2}{2}$

$=38.4$ m/s (up)

b) Determine the **approximate** instantaneous velocity at t = 2, using appropriate secant approximations

$2\leq t\leq 2.01$

$IROC=\frac{40(2.01)-0.8(2.01)^2-(40(2)-0.8(2)^2)}{0.01}$

$\doteq 36.8$ m/s

Unit 2 Rational Functions

1. State the domain, the x and y-intercepts and the equations of any asymptotes for each of the following.

	$f(x)=\frac{x-2}{x}$	$g(x)=\frac{x^2-2x-15}{x+2}$
Domain	$\{x\in\mathbb{R} \mid x\neq 0\}$	$\{x\in\mathbb{R} \mid x\neq -2\}$
x-intercept $y=0$	$x=2$ $(2,0)$	$x=-3,\ 5$ $(-3,0)(5,0)$
y-intercept	NONE	$-\frac{15}{2}$ $(0,-\frac{15}{2})$
Horizontal Asymptote	$y=1$	None (oblique asymptote)
Vertical asymptote	$x=0$	$x=-2$

2. Determine the point(s) of intersection of the following two rational functions, **algebraically**.

$$f(x)=\frac{x^2}{9x+4} \text{ and } g(x)=\frac{2}{2x-3}$$

$\frac{x^2}{9x+4}=\frac{2}{2x-3}$ cross multiply

$2x^3-3x^2-18x-8=0$

3. The volume V in litres of a sample of hydrogen chloride gas is given by the formula $V=\frac{280}{p}$, where p is the pressure in kilopascals. Determine the rate of change of volume when:

a) pressure changes from 10 kilopascals to 20 kilopascals

AROC

$AROC=\frac{V(20)-V(10)}{20-10}$

$=\frac{14-28}{10}$

$=-1.4$ L/k

b) the pressure is 20 kilopascals $20\le p\le 20.01$

$IROC=\frac{V(20.01)-V(20)}{20.01-20}$

$=-0.7$ L/k

4. A motor boat coasts towards a dock with its engine off. Its distance s (in metres) from the dock, t seconds after the engine is cut off is : $s(t)=\frac{10(6-t)}{3+t}$

a) How far is the boat from the dock initially? $t=0$

$s(0)=20$ m

b) How long does it take for the boat to reach the dock?

$t=6$ seconds

$(s(t)=0)$

c) Determine the boat's velocity when it bumps into the dock $6 \le t \le 6.01$

d) Explain the graphical significance of each of the above values

FACTOR

5. Provide a sketch of the following functions (on separate grids):

a) $f(x)=\frac{x-2}{3x-4}$ x-int: 2 y-int: $\frac{1}{2}$

HA: $y=\frac{1}{3}$

end beh

$x\to\infty$ $y\to\frac{1}{3}^-$

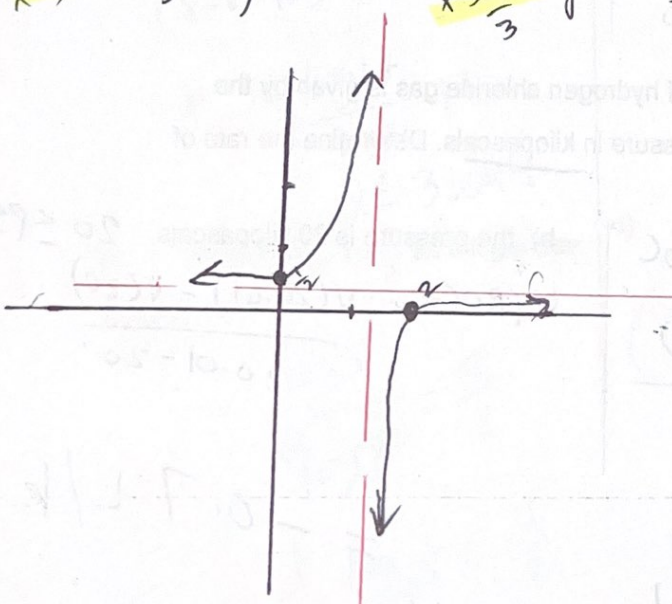
$x\to-\infty$ $y\to\frac{1}{3}^+$

VA: $x=\frac{4}{3}$

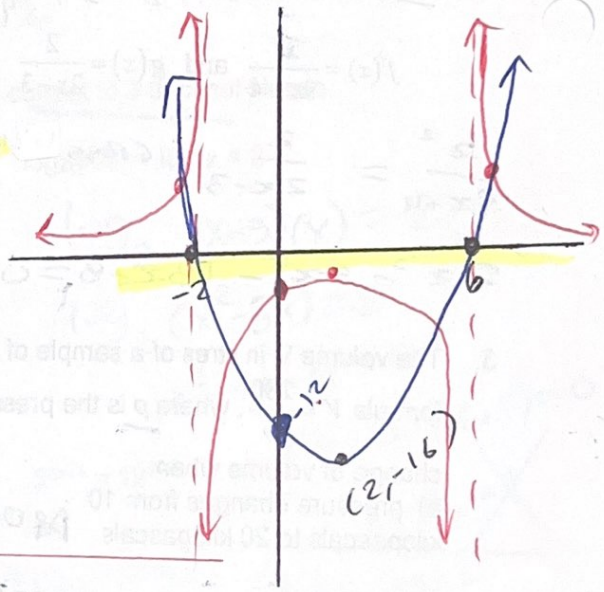
beh VA

$x\to\frac{4}{3}^+$ $y\to-\infty$

$x\to\frac{4}{3}^-$ $y\to\infty$



b) $g(x)=\frac{1}{x^2-4x-12}=\frac{1}{(x-6)(x+2)}$



$y>0$, $y\le\frac{-1}{16}$

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Unit 3 Exponential and Logarithmic Functions

1. Evaluate each of the following. Exact answers are necessary.

a) $\frac{6^{\log_6 14}}{7^{\log_7 15}} = \frac{14}{15}$

b) $\left(\frac{125}{64}\right)^{\frac{-2}{3}} = \left(\frac{64}{125}\right)^{\frac{2}{3}} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

c) $\log_3 27^4 = 4 \log_3 27$

$= 12$

d) $\frac{\log_5 81}{\log_5 3} = \log_3 81 = 4$

2. Evaluate each of the following. Exact solutions are required.

a) $4\log_3 3 + \log_3 4 - \log_3 12$

$= \log_3 \frac{3^4 \cdot 4}{12}$

$= 3$

b) $\log_4 \sqrt{4^{-1}} = \frac{1}{2}\log_4 2$

$= -\frac{1}{2} - 3\left(\frac{1}{2}\right) = -2$

3. Solve the following. Where necessary, answers correct to 3 decimal places.

a) $4^{x-1} = 17$

$(x-1)\log 4 = \log 17$

$x = \frac{\log 17}{\log 4} + 1$

$x \doteq 3.044$

b) $\log_2(x-3) + \log_2 x = 2$

$\log_2 (x-3)(x) = 2$

$\log_2 (x^2 - 3x) = 2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

c) rewrite $\log_{27} 5 = \frac{a}{2}$ as a logarithm of base 3

$\frac{\log 5}{\log 27} = \frac{a}{2}$

$\frac{1}{3}\log_3 5 = \frac{a}{2}$

d) $5^{3x+1} = 12^{x+4}$

$(3x+1)\log 5 = (x+4)\log 12$

$3x\log 5 + \log 5 = x\log 12 + 4\log 12$

$x(3\log 5 - \log 12) = 4\log 2 - \log 5$

$x = \frac{\log \frac{16}{5}}{\log \frac{125}{12}}$

$A = a(2)^{t/d}$ $\quad A = a\left(\frac{1}{2}\right)^{t/h}$

a A t

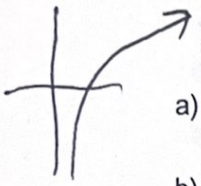
4. A bacteria colony grows exponentially from 134 cells to 1241 cells in 24 hours. What is the doubling period, t, in hours and minutes? $d = ?$

$$\frac{1241}{134} = \frac{134(2)^{\frac{24}{d}}}{134} \Rightarrow \frac{24}{d} = \frac{\log \frac{1241}{134}}{\log 2}$$

$d = 7.47$ hours

OR 448.4 mins

5. For the function $f(x) = \log_2 x$ state:



a) f(1) 0

b) the growth behaviour of f(x) increasing/up.

c) the equation for the asymptote $x = 0$

h a

6. The half-life of CO^{60} is 5.24 years. Determine how long it takes for 2 mg of CO^{60} to decay to 0.2 mg. (to 2 decimal places)

A

$$\frac{0.2}{2} = \frac{2\left(\frac{1}{2}\right)^{\frac{t}{5.24}}}{2}$$

$$\frac{\log 0.1}{\log 0.5} = \frac{t}{5.24} \Rightarrow t = 17.41 \text{ years}$$

7. Describe the **translation(s)** on the graph of $y = \log_4 x$ required to produce the graph of $y = \log_4\left(\frac{x}{16}\right)$. Explain referencing the appropriate property/properties of logarithms.

$= \log_4 x - \log_4 16$

$= \log_4 x - 2$

8. Simplify the following using cofunction identities:

$$\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}+x\right)\cos\left(\frac{\pi}{2}-x\right)$$

$\sin(a-b)$
$\cos(a+b)$
$\cos(a-b)$

$= \cos x - (\sin x)(\sin x)$

$= \cos x - \sin^2 x$

9. Given the equation $y = \frac{1}{3}\cos(2x-\pi)-5$, state the following:

a) amplitude $\frac{\max - \min}{2} = 1/3$

b) vertical translation $y = (\max+\min)/2$ 5 units down

c) phase shift $\pi/2$ to the right

d) period $\frac{2\pi}{k}$ π

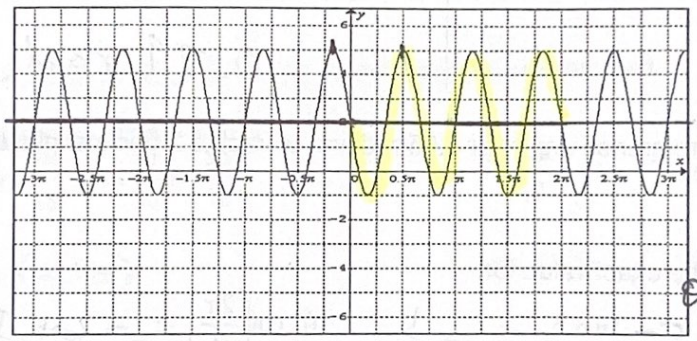
e) domain D = $\{x \in \mathbb{R}\}$ / $\{x \in \mathbb{R} \mid 0 \le x \le \pi\}$ FOR ONE CYCLE

f) range R = $\{y \in \mathbb{R} \mid -5\frac{1}{3} \le y \le -4\frac{2}{3}\}$

$\max = c + |a|$ $\min = c - |a|$

10. Determine the equation for the following graph if the basic function was cosine.

$y = 3\cos[3(x-\pi/2)]+2$



$a = 3$.
$c = y = 2$
$k = 3$

$\theta = \pi - \alpha$ | $\theta = \alpha$
$\theta = \pi + \alpha$ | $\theta = 2\pi - \alpha$

11. Solve each equation. Find all possible solutions given $0 \le x \le 2\pi$

a) $2\sin x + \sin x \cos x = 0$

$\sin x(2+\cos x) = 0$

$\sin x = 0$ | $\cos x = -2$ soln not possible.

$x = 0, \pi, 2\pi$

b) $2\tan^2 x + 3\tan x + 1 = 0$

5.82, 2.68, $\frac{3\pi}{4}$, $\frac{7\pi}{4}$

$(2\tan x + 1)(\tan x + 1) = 0$

$\tan x = -1/2$ | $\tan x = -1$

Q2, Q4 | Q2, Q4

$\pi - 0.46$, $2\pi - 0.46$ | $\pi - \frac{\pi}{4}$, $2\pi - \frac{\pi}{4}$

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Unit 4 Trigonometric Functions

1. Convert to radian measure:
   a) exact values.
   b) Correct to 2 decimal places.

   a) $75° \times \frac{\pi}{180} = \frac{5\pi}{12}$ rad

   b) $319° \times \frac{\pi}{180} \doteq 5.57$ rad.

2. Convert to degrees. (Round your answer to the nearest tenth of a degree.)

   a) $\frac{\pi}{7} = \frac{180°}{7} = 25.7°$

   b) $\frac{7\pi}{13} = 96.9°$

3. If $0 \le A \le 2\pi$ find $\angle A$ if :

   a) $\sin A = \frac{-\sqrt{3}}{2}$

   Q3: $A = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

   Q4: $A = 2\pi - \pi/3 = 5\pi/3$

   b) $\cos A = \frac{1}{\sqrt{2}}$

   Q1: $A = \pi/4$

   Q4: $A = 2\pi - \frac{\pi}{4} = 7\pi/4$

4. Express the angle that is formed when the hands on a clock indicate it is 7:30 p.m.

   a) in degree measure

   $45°$

   b) in radian measure (exact)

   $\pi/4$

   c) In radian measure (approximate, to the nearest hundredth of a radian)

   0.79 rad.

5. What is the reference angle for 3.85 radians correct to 2 decimal places?

6. Determine the exact value for:

   a) $\sin\left(\frac{5\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} = -\sqrt{2}/2$

   b) $\cot\left(\frac{2\pi}{3}\right) = -\cot\frac{\pi}{3} = -\tan\pi/6 = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

7. Solve the following. An exact answer is required.

$$\frac{\csc\frac{\pi}{3} + \cot\frac{\pi}{6}}{2 - \sec 2\pi} = \frac{\frac{1}{\sin\pi/3} + \tan\frac{\pi}{3}}{2 - \frac{1}{\cos 2\pi}} = \frac{\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{1}}{2-1} = \frac{2+3}{\sqrt{3}}$$

$$= \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

12. Prove the following identities:

a) $\frac{\cos(x-y)}{\sin x \cos y} = \cot x + \tan y$

$LS = \frac{\cos x \cos y + \sin x \sin y}{\sin x \cos y}$

$= \cot x + \tan y$

b) $\frac{1}{\csc x - \sin x} = \sec x \tan x$

$LS = \frac{1}{\csc x - \sin x}$

$= \frac{1}{\frac{1}{\sin x} - \frac{\sin x}{1}} = \frac{1}{\frac{1-\sin^2 x}{\sin x}}$

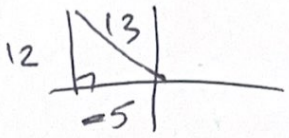
$= \frac{\sin x}{1-\sin^2 x}$

$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x \cos x}$

$= \tan x \sec x$

$= RS$

13. If $\csc B = \frac{13}{12}$; $\frac{\pi}{2} < B < \pi$. determine the value of tan *B*.



$\sin B = \frac{12}{13}$

$\tan B = \frac{opp}{adj} = \frac{12}{5}$

14. Evaluate using the addition formulae, exact answers are required.

a) $\sin\frac{5\pi}{36}\cos\frac{5\pi}{18} + \cos\frac{5\pi}{36}\sin\frac{5\pi}{18}$

$\sin a \cos b \quad \cos a \sin b$

$= \sin(a+b)$

$= \sin\left(\frac{5\pi}{36} + \frac{5\pi}{18}\right)$

$= \sin\left(\frac{15\pi}{36}\right)$

$= \sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

b) $\sin\frac{11\pi}{12} = \sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$